Math 223 01
Prof. Doug Baldwin

Problem Set 10 — Partial Derivatives

Complete by **Tuesday, March 31**
Grade by **Thursday, April 2**

Purpose

This problem set reinforces your understanding of partial derivatives and tangents to higher-dimensional surfaces. By the time you finish this problem set, I expect that you will be able to

- Calculate higher-order partial derivatives
- Use Mathematica to find partial derivatives
- Find linear approximations to multivariable functions
- Use the chain rule for partial derivatives.

Background

This exercise is based on the latter part of section 13.3 in our textbook, and on sections 13.4 and 13.5. We covered these sections in classes between March 6 and 11.

Activity

Solve the following problems.

Question 1. Here is a function of 3 variables:

$$f(x, y, z) = xy^2z + x\cos z - z\ln(x + y)$$

Part A. Find all the first and second partial derivatives of this function.

Part B. Use Mathematica to verify your answers to Part A.

Part C. Based on your results from the first 2 parts, how do you think Clairaut's Theorem (the mixed-derivative theorem) applies to functions of 3 variables?

Question 2. Calculate the value of $f(x, y, z) = \sqrt{4 - x^2 - y^2 - z^2}$ at $(x, y, z) = (1, 1, 1)$. Then use a linear approximation to estimate the value at $(1.01, 0.98, 1.02)$. Finally, calculate the actual value at $(1.01, 0.98, 1.02)$ and compare it to your estimate. You may use a calculator for the numeric calculations in this problem.

Notice that you will need a 3-variable version of linear approximation, for which you will have to invent or find a formula. Be prepared during your grading meeting to explain why the formula you found or came up with makes sense.

Question 3. (Based on exercise 4 in section 13.5 of our textbook.) Suppose $w = xy^2$, and that $x = 5\cos(2t)$ and $y = 5\sin(2t)$. Use the chain rule to find $\frac{dw}{dt}$. Then substitute the definitions of x and y into the equation for w, to get an equation in terms of t from which you can calculate $\frac{dw}{dt}$ directly. Verify that both ways of calculating the derivatives produce the same answer.

Question 4. (Exercise 38 in section 13.5 of the textbook.)

The equation

$$PV = kT$$

relates the pressure (P), volume (V) and temperature (T) of a gas. Find $\frac{dP}{dt}$ given information about V, T, and their derivatives with respect to time (t). See the textbook for the details, but treat temperature as kelvins, not degrees Fahrenheit.

Follow-Up

I will grade this exercise in a video meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please have a written solution to the exercise available during your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you worked in a group on this assignment and the group has one collective solution, the whole group can schedule a single meeting with me.