Math 223 01
Prof. Doug Baldwin

Problem Set 1 — 3-Dimensional Coordinate Systems

Complete by **Wednesday, January 29**
Grade by **Friday, January 31**

Purpose

This problem set reinforces your understanding of 3-dimensional coordinate systems and of plotting 3-dimensional implicit equations with Mathematica. Specifically, by the time you finish this problem set I expect you to be able to...

- Relate positions and regions in space to coordinates in a 3-dimensional coordinate system
- Calculate and otherwise reason about distances in three or more dimensions
- Reason about equations for simple shapes (lines, planes, spheres, etc.) in space
- Plot simple shapes in space with Mathematica.

Background

Most of this problem set is based on the first half of section 11.2 in our textbook. We discussed that material in class on January 24.

Plotting with Mathematica is not covered in our textbook, but we discussed it as part of a general introduction to Mathematica in class on January 27.

Activity

Solve the following problems:

Question 1. Use Mathematica to plot the equation $x^2 - 1 = 0$ over the 3-dimensional region $-2 \le x \le 2, -2 \le y \le 2, -2 \le z \le 2$. Explain in English why this equation produces the surface(s) you see in Mathematica.

Question 2. Remember the computer animated snake from our introductory class? Now that we understand 3-dimensional coordinate systems, we can start talking about positions of parts of it. For example, suppose the tip of the snake's tail is at point $(-10, 1, 0)$ in some coordinate system, and that its body twists around in the $z = 0$ plane to the origin. At the origin, the snake's neck abruptly goes straight up for 2 units, to the center of the snake's head.

Part A. What are the coordinates of the center of the snake's head?

Part B. How far is the center of the head from the tip of the tail (in a straight line, not along the snake's neck and body)?

Part C. Suppose the snake's head is a sphere 1 unit in radius. Give an equation for the head. Then plot that equation with Mathematica to verify that it is indeed a sphere in the expected place (hint: pick a region to plot over that will make it easy to see the whole shape and where it is).

Question 3. Consider the points in 3 dimensional space that satisfy the constraints $-1 \leq x \leq 1$, $y \geq 0$, and $z = 3$.

Part A. Describe the region that contains these points in English and/or with a diagram.

Part B. For each of the following points, say whether or not it is in the region and why or why not:

- $(0, 3, 1)$
- $(0, 1, 3)$
- $(1, -1, 3)$

Part C. Identify one point not listed in Part B that is in the region.

Question 4. Renowned time traveler Dr. Whowhatwhenwherewhyandhow travels from spacetime point $(1, 3, 2, 6)$ to $(-1, 5, 4, 4)$. How far has the Doctor traveled? (Assume time travelers measure space and time in units of mileyears — being merely 3-dimensional creatures, we can't really visualize what a mileyear looks like, but somehow it is a unit that manages to apply equally to "distance" in space and time. So, for example, you can interpret the Doctor's starting point as being 1 mileyear from the origin in the x direction, 3 mileyears in the y, 2 mileyears in the z, and 6 in the time direction; the question is asking how many mileyears the Doctor has traveled.)

Question 5. Our textbook leaves the proof of the 3-dimensional distance formula as an exercise (although I don't actually see it among the exercises for section 11.2). We sketched the proof in class on January 24, although with some vague references to using the Pythagorean Theorem or 2-dimensional distance formula on certain triangles in certain planes. Flesh out this argument, particularly giving the details of how the Pythagorean Theorem fits in. You may find Figure 11.2.5 in the book, and its terminology, helpful.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you worked in a group on this assignment and the group has one collective solution, the whole group can schedule a single meeting with me.