Problem Set 9 — Integrating Multivariable Functions

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Math 223 04

Complete By Wednesday, April 18
Grade By Monday, April 23

Purpose

This problem set develops your ability to evaluate integrals of multivariable functions and to use them to model physical situations.

Background

This exercise is based on material from sections 5.1 through 5.4, plus section 5.6, of our textbook. We discussed this material in classes between March 30 and April 9 (but note that we didn't discuss it in quite the same order as it's presented in the text; in particular we treated double and triple integrals as two forms of a single idea whereas the text treats them separately).

Activity

Solve the following problems.

Problem 1. (Exercise 100 in section 5.2 of OpenStax *Calculus Volume 3.*)

Evaluate

$$\iint_D (x^2 + y)\, dA$$

where region D is the region between the curves $y = -4 + x^2$ and $y = 4 - x^2$. See the book for a sketch of the region.

Check your answer by evaluating the integral with muPad or a similar tool.

Problem 2. (Exercise 150 in section 5.3 of OpenStax *Calculus Volume 3.*)

Convert the integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} (x + y)\, dy\, dx$$

to polar coordinates and evaluate.

Check your result by using muPad (or a similar tool) to evaluate the original integral in rectangular coordinates.

Problem 3. Evaluate

$$\int_0^3 \int_0^2 \int_0^1 \int_0^z dw\, dz\, dy\, dx$$

Give an interpretation of this integral as a "volume" (or, technically, hypervolume) that helps you make sense of the value you calculated.

Problem 4. (Exercise 338 in section 5.6 of OpenStax *Calculus Volume 3.*)

Find the mass, moments, and center of mass of a shape with density $\rho(x, y, z) = x + y + 1$ and bounds $0 \leq x \leq 1$, $0 \leq y \leq 2$, and $0 \leq z \leq 3$. Use muPad or a similar tool to do the necessary integrations and other calculations.

Problem 5. (Exercise 164 in section 5.3 of OpenStax *Calculus Volume 3.*)

Find the volume of a shape (a "spherical ring") created by boring a 2-inch radius cylinder through the center of a sphere of radius 4 inches. See the book for a picture of the shape and further discussion. Note that the volume I want is the volume remaining from the sphere after removing the cylinder (not the volume of the cylinder).

Note: this problem has a relatively straightforward solution via the "washer method" from single-variable calculus, and a more intricate solution via multiple integrals. I expect you to be able to solve it somehow, but the multiple integral solution is challenging enough that doing it well goes a little beyond what I expect, especially if you do it and some other method and can compare the two.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.

I will use the following guidelines in grading this problem set:

- What I expect (8 points). Your written solutions and verbal explanations show that you understand (1) how to evaluate multiple integrals, (2) how to transform multiple integration problems in rectangular coordinates to polar and evaluate, (3) how to express such quantities as volume, mass, or moment as multiple integrals, and (4) how to evaluate multiple integrals with muPad or similar technology. See the note at the end of problem 5 for information about what I expect for it specifically.

- Half of what I expect (4 points). Plausible but non-exclusive examples include showing that you understand 2 of the expected items, with no understanding of the others, OR showing that you partially but not completely understand all the expected items.

- Exceeding what I expect (typically 1 point added to what you otherwise earn). Generally, demonstrating that you have nontrivially engaged with math in ways beyond what is needed to solve the given problems exceeds my expectations. Also see the note at the end of problem 5 for expectations specific to that problem.