Problem Set 8 — Extreme Values and Optimization

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Math 223 04

Complete By Monday, April 9
Grade By Thursday, April 12

Purpose

This problem set develops your ability to find extreme values and saddle points of multivariable functions, and to use Lagrange multipliers to solve optimization problems. As a secondary goal, this problem set begins to develop your ability to evaluate definite integrals of multivariable functions.

Background

This exercise is based on material from sections 4.7, 4.8, and 5.1 of our textbook. We discussed this material in classes between March 21 and 30.

Activity

Solve the following problems.

Problem 1. (Exercise 344 in section 4.7 of OpenStax *Calculus Volume 3*.)

Find the absolute extrema of function $f(x, y) = xy - x - 3y$ over the triangular region with vertices $(0, 0)$, $(0, 4)$, and $(5, 0)$. You may use a calculator to evaluate the numeric expressions that arise in solving this problem.

Finally, use muPad to plot function $f(x, y)$ to check visually that your calculated answers make sense.

Problem 2. Find all the saddle points of the function $f(x, y) = x^4 - 3x^2 - y^2$. Verify that the second derivative test correctly picks these points out from the other critical points. Finally, use muPad to plot the function so you can check visually that your calculated answers make sense.

Problem 3. (Exercise 358 in section 4.8 of OpenStax *Calculus Volume 3*.)

Find the minimum and maximum values that function $f(x, y) = x^2 y$ takes on subject to the constraint $x^2 + 2y^2 = 6$.

Problem 4. (Exercise 388 in section 4.8 of OpenStax *Calculus Volume 3.*)

A large container in the shape of a rectangular solid must have a volume of 480 cubic meters. The bottom of the container costs \$5 per square meter to construct; the top and sides cost \$3 per square meter. Use Lagrange multipliers to find the dimensions of the container of this size that has the minimum cost.

You may use a calculator for the numeric calculations in this exercise.

Problem 5. In class on March 30, someone pointed out that

$$\int_1^2 \int_0^1 \int_0^2 xy^2 + x^2 z \, dx \, dy \, dz = \int_0^1 \int_0^2 xy^2 \, dx \, dy + \int_1^2 \int_0^2 x^2 z \, dx \, dz$$

and asked if this illustrated a general way to break certain high-dimension integrals into lower-dimension ones. I said that it doesn't, that the equation only worked for this example because the intervals being integrated over in the y and z dimensions happened to be of length 1.

Demonstrate my claim by calculating

$$\int_1^3 \int_0^3 \int_0^2 xy^2 + x^2 z \, dx \, dy \, dz$$

and showing that you get a different answer from what you get by calculating

$$\int_0^3 \int_0^2 xy^2 \, dx \, dy + \int_1^3 \int_0^2 x^2 z \, dx \, dz$$

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.

I will use the following guidelines in grading this problem set:

- What I expect (8 points). Your written solutions and verbal explanations show that you understand (1) how to find and interpret critical points of multivariable functions, (2) how to find absolute extremes over closed regions, (3) how to use Lagrange multipliers to solve optimization problems, and (4) how to evaluate definite integrals of multivariable functions.

- Half of what I expect (4 points). Plausible but non-exclusive examples include showing that you understand 2 of the expected items, with no understanding of the others, OR showing that you partially but not completely understand all the expected items.

- Exceeding what I expect (typically 1 point added to what you otherwise earn). Exploring integration in muPad (in addition to doing it manually) would be one way to exceed my expectations for this problem set. Demonstrating that you have nontrivially engaged with math in other ways beyond what is needed to solve the given problems also exceeds my expectations.