Problem Set 7 — More about Partial Derivatives

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Math 223 04

Complete By Sunday, April 1
Grade By Wednesday, April 4

Purpose

This problem set develops your ability to reason in relatively sophisticated ways about partial derivatives, e.g., to use the chain rule with partial derivatives or to work with directional derivatives.

Background

This exercise is based on material from sections 4.5 and 4.6 of our textbook. We discussed this material in classes on March 8 (the chain rule), March 9 (directional derivatives) and March 19 (gradients).

Activity

Solve the following problems.

Problem 1. (Based on exercise 216 in section 4.5 of OpenStax *Calculus Volume 3*.)
Let $w(t, v) = e^{tv}$, where $t = r + s$ and $v = rs$.
Part 1. Use the chain rule to find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial r}$.
Part 2. Substitute the definitions of t and v into the definition of w and find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial r}$ directly; verify that the derivatives found this way are the same as the ones found in Part 1.

Problem 2. (Exercise 252 in section 4.5 of OpenStax *Calculus Volume 3*.)
Starting with the equation

$$PV = kT$$

that relates the pressure (P), volume (V) and temperature (T) of a gas, find $\frac{dV}{dt}$ given information about P, T, and their derivatives. See the textbook for the details.

Problem 3. (Exercise 272 in section 4.6 of OpenStax *Calculus Volume 3*.)
Find the directional derivative at $(5, 5)$ of $f(x, y) = x^2y$ in direction $\langle 3, -4\rangle$.

Problem 4. Let $z = 4 - x^2 - y^2$. Show that no matter what direction you move from $(x, y) = (0, 0)$, the instantaneous rate of change in z is 0.

Problem 5. Find the gradient of $f(x, y, z) = \ln(x^2y + y^3z + zx)$ in terms of x, y, and z. Then find the numeric value of the gradient at point $(1, 2, 3)$.

Problem 6. (Exercise 306 in section 4.6 of OpenStax *Calculus Volume 3*.)
The temperature in a metal sphere is inversely proportional to distance from the origin. Show that at any point in the sphere the direction of greatest increase in temperature is towards the origin, and then find the rate of change in temperature at point $(1, 2, 2)$ in the direction towards point $(2, 1, 3)$. (See the textbook for a different wording of this problem.)

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.

I will use the following guidelines in grading this problem set:

- What I expect (8 points). Your written solutions and verbal explanations show that you understand (1) how to use the chain rule to find partial derivatives, (2) how to find directional derivatives, (3) how to interpret directional derivatives as rates of change, (4) how to find gradients, and (5) how to use gradients as problem-solving tools.

- Three quarters of what I expect (6 points). A plausible but not exclusive example is showing that you fully understand 4 of the expected items and partially or completely fail to understand the remaining one.

- Half of what I expect (4 points). Plausible but non-exclusive examples include showing that you understand 2 or 3 of the expected items, with no understanding of the others, OR showing that you partially but not completely understand all the expected items.

- Exceeding what I expect (typically 1 point added to what you otherwise earn). One, but not the only, plausible way of exceeding expectations for this problem set is to produce muPad (or similar) visualizations of some of the situations in the questions. Demonstrating in other ways that you have significantly engaged with math beyond what is needed to solve the given problems also counts as exceeding expectations.