Problem Set 6 — Partial Derivatives

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Math 223 04

Complete By Wednesday, March 21
Grade By Monday, March 26

Purpose

This problem set mainly develops your ability to reason about partial derivatives and some of their applications. It also gives you more practice working with limits of multivariable functions.

Background

Our textbook discusses partial derivatives in section 4.3. We discussed partial derivatives in class on March 5.

The textbook discusses tangent planes and related ideas (e.g., linear approximations, differentials) in section 4.4. We discussed this material in class on March 7.

Finally, our book discusses limits of multivariable functions in section 4.2. We discussed them in class on March 1.

Activity

Solve the following problems.

Problem 1. (Exercise 76 in section 4.2 of OpenStax *Calculus Volume 3*.)
Determine whether

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$$

exists, and if so what it is.

Problem 2. Find all the (first) partial derivatives of each of the following functions:

1. $f(x,y) = \sin(xy) - xy$
2. $g(x,y,z,w) = \frac{x^2-y^2}{z^2+w^2}$

Problem 3. Find all the second partial derivatives (including mixed derivatives) of $f(x, y) = \sin(xy) - xy$. Note that this is one of the functions from Problem 2, so your solution to that problem will give you first derivatives to start from.

Problem 4. (Exercise 132 in section 4.3 of OpenStax *Calculus Volume 3.*)
The area of a parallelogram with adjacent sides of length a and b that meet at angle Θ is $A(a, b, \Theta) = ab \sin \Theta$. Find the rate of change of the area of the parallelogram with respect to each variable a, b, and Θ.

Problem 5. (Inspired by exercise 198 in section 4.4 of OpenStax *Calculus Volume 3.*)
Calculate the value of $z = \sqrt{4 - x^2 - y^2}$ at $(x, y) = (1, 1)$. Then use a linear approximation to estimate the value at $(1.01, 0.97)$. Finally, calculate the actual value at $(1.01, 0.97)$ and compare it to your estimate. You may use a calculator for the numeric calculations in this problem.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.

I will use the following guidelines in grading this problem set:

- What I expect (8 points). Your written solutions and verbal explanations show that you understand (1) how to find limits of multivariable functions, (2) how to find partial derivatives, (3) how to find second or other higher-order partial derivatives, (4) how to interpret partial derivatives as rates of change or slopes of tangents, and (5) how to find and use a linear approximation to a multivariable function.

- Three quarters of what I expect (6 points). A plausible but not exclusive example is showing that you fully understand 4 of the expected items and partially or completely fail to understand the remaining one.

- Half of what I expect (4 points). Plausible but non-exclusive examples include showing that you understand 2 or 3 of the expected items, with no understanding of the others, OR showing that you partially but not completely understand all the expected items.

- Exceeding what I expect (typically 1 point added to what you otherwise earn). One, but not the only, plausible way of exceeding expectations for this problem set is to produce muPad (or similar) visualizations of some of the functions in the questions. Demonstrating in other ways that you have significantly engaged with math beyond what is needed to solve the given problems also counts as exceeding expectations.