Problem Set 5 — Curvature and Arc Length

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Math 223 04

Complete By Monday, March 5
Grade By Friday, March 9

Purpose

This problem set mainly develops your ability to work with and reason about curvature and arc length of vector valued functions. This problem set also begins to develop your understanding of multivariable functions and their limits.

Background

Our textbook discusses arc length and curvature in section 3.3. We discussed arc length in class on February 14, and curvature on February 15.

Our text discusses multivariable functions in section 4.1, and their limits in section 4.2. We discussed (or will discuss) that material in class on February 28 and March 1.

Activity

Solve the following problems.

Problem 1. (Inspired by exercise 107 in section 3.3 of OpenStax *Calculus Volume 3.*)
Find the length of one turn of the helix $\vec{r}(t) = \langle 2\sin t, \sqrt{5}t, 2\cos t\rangle$.

Problem 2. An ant crawls along the curve $\vec{r}(t) = \langle 2t^2, t^2 - 1, \frac{\sqrt{5}}{2}t^2\rangle$, starting at $\vec{r}(1)$. What point is the ant at after it has crawled 1 unit in the direction of increasing t?

Problem 3. Find a unit vector that points in the direction the curve $\vec{r}(t) = \langle \cos(e^t), \sin(e^t), 0\rangle$ is turning when $t = \ln \pi$.

Problem 4. (Based on exercise 14 in section 4.1 of OpenStax *Calculus Volume 3.*)
Give an equation for, and sketch, the level curves for $c = 1$ of the function $z(x, y) = y^2 - x^2$.

Problem 5. (Inspired by exercise 86 in section 4.2 of OpenStax *Calculus Volume 3.*)
Show that

$$\lim_{(x,y)\to(0,0)} \frac{xy+y^3}{x^2+y^2}$$

does not exist.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.

I will use the following guidelines in grading this problem set:

- What I expect (8 points). Your written solutions and verbal explanations show that you understand (1) how to find arc length of a vector valued function, (2) how to find and use the arc length parameterization of a vector valued function, (3) how to use curvature-related information to describe the way a curve turns, (4) what level curves are and how to find them, and (5) how to determine whether a limit of a multivariable function exists,
- Three quarters of what I expect (6 points). A plausible but not exclusive example is showing that you fully understand 4 of the expected items and partially or completely fail to understand the remaining one.
- Half of what I expect (4 points). Plausible but non-exclusive examples include showing that you understand 2 or 3 of the expected items, with no understanding of the others, OR showing that you partially but not completely understand all the expected items.
- Exceeding what I expect (typically 1 point added to what you otherwise earn). One, but not the only, plausible way of exceeding expectations for this problem set is to produce muPad (or similar) visualizations of some of the curves or functions in the questions. Demonstrating in other ways that you have significantly engaged with math beyond what is needed to solve the given problems also counts as exceeding expectations.