Problem Set 4 — Vector Valued Functions

Prof. Doug Baldwin

Math 223 04

Complete By Sunday, February 25
Grade By Wednesday, February 28

Purpose

This problem set develops your ability to work with vector valued functions, their derivatives, and antiderivatives.

Background

This problem set is based on material from sections 3.1 and 3.2 of our textbook. We discussed, or will discuss, this material in class between February 7 and 12.

Activity

Solve the following problems.

Problem 1. Let $\vec{r}(t) = \langle t\cos t, 2t\sin t, t^2\rangle$.

Part 1. Find $\vec{r}(\pi)$ and $\vec{r}(2)$.

Part 2. Use muPad or similar technology to plot $\vec{r}(t)$ over the interval $-\pi \leq t \leq \pi$.

Part 3. Does $\vec{r}(t)$ pass through the origin? If so, at what value of t? (I expect a calculated answer to this question, not one based only on graphs).

Problem 2. Since vector valued functions produce vectors, you can also compute the dot product of two vector valued functions.

Part 1. Find the dot product $\vec{f}(t) \cdot \vec{g}(t)$ where $\vec{f}(t) = \langle t\sin t, t, 3t^2\rangle$ and $\vec{g}(t) = \langle \sin t, \cos^2 t, \frac{1}{t}\rangle$.

Part 2. Show that for any 3-dimensional vector valued functions $\vec{f}(t)$ and $\vec{g}(t)$ and constant c, such that $\lim_{t\to c}\vec{f}(t)$ and $\lim_{t\to c}\vec{g}(t)$ exist, $\lim_{t\to c}(\vec{f}(t)\cdot\vec{g}(t)) = (\lim_{t\to c}\vec{f}(t))\cdot(\lim_{t\to c}\vec{g}(t))$.

Problem 3. Let $\vec{r}(t) = \langle \sin(e^t), \cos(e^t), e^t\rangle$.

Part 1. Find the derivative of $\vec{r}(t)$ with respect to t.

Part 2. Find the second derivative of $\vec{r}(t)$ with respect to t.

Part 3. Find the unit tangent vector to $\vec{r}(t)$.

Problem 4. (Problem 42 in section 3.2 of OpenStax *Calculus Volume 3*.)

An object starts at rest at point $(1, 2, 0)$ and accelerates with acceleration $\vec{a}(t) = \langle 0, 1, 2 \rangle$ (with magnitude measured in feet per second per second). Find the location of the object after 2 seconds.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.

I will use the following guidelines in grading this problem set:

- What I expect (8 points). Your written solutions and verbal explanations show that you understand (1) how a vector valued function defines a set of points or vectors in space, (2) how to graph vector valued functions with muPad or similar technology, (3) limits of vector valued functions, (4) derivatives of vector valued functions, and (5) antiderivatives of vector valued functions.

- Three quarters of what I expect (6 points). A plausible but not exclusive example is showing that you fully understand 4 of the expected items and partially or completely fail to understand the remaining one.

- Half of what I expect (4 points). Plausible but non-exclusive examples include showing that you understand 2 or 3 of the expected items, with no understanding of the others, OR showing that you partially but not completely understand all the expected items.

- Exceeding what I expect (typically 1 point added to what you otherwise earn). Some, but not the only, plausible ways of exceeding expectations for this problem set include additional plotting or other visualization beyond what I ask for, or doing the proof for Problem 1 for every number of dimensions, not only 3. Demonstrating in other ways that you have significantly engaged with math beyond what is needed to solve the given problems also counts as exceeding expectations.