Problem Set 3 — Lines and Planes

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Math 223 04

Complete By Thursday, February 8
Grade By Tuesday, February 13

Purpose

This problem set reinforces your understanding of lines and planes in space, and briefly exercises your understanding of spherical coordinates.

Background

Lines and planes come from Section 2.5 of our textbook. We covered that material in class on February 1 and 2.

Section 2.7 of the textbook discusses spherical coordinates. We covered that material in class on February 5.

Activity

Solve the following problems.

Problem 1. (Problem 248 from section 2.5 of of OpenStax *Calculus Volume 3.*)

Find parametric and symmetric equations for the line through point $(3, 1, 5)$ in direction $\langle 1, 1, 1\rangle$. Also find the point at which this line intersects the xy plane.

Problem 2. (Based on problem 268 from section 2.5 of of OpenStax *Calculus Volume 3.*)

Find an equation for the plane that contains point $(3, 2, 2)$ and has normal vector $\langle 2, 3, -1\rangle$.

Problem 3. (This is the problem that we set up in class on February 1, but didn't have time to calculate the answer to.)

Fearless (but not very competent) space traveler Spacey is fleeing an evil alien starbase. Spacey is making for planet G'nseo. The vector from the alien starbase to G'nseo is $\langle 8, 2, -2\rangle$. Unfortunately, Spacey is actually moving in direction $\langle 5, 1, 0\rangle$. How close to G'nseo does Spacey get?

Problem 4. Part 1. Given a line, $\vec{r}(t) = P + t\vec{v}$ and a plane $ax + by + cz = d$, explain a test you could use to determine whether the line is parallel to the plane.

Part 2. Use your test to determine whether the following lines are parallel to plane $x + y + z = 1$:

1. $\vec{r}(t) = (1, 0, 0) + t\langle 2, 1, 0\rangle$
2. $\vec{r}(t) = (2, 1, 2) + t\langle 1, 0, -1\rangle$

Part 3. (Problem 276 from section 2.5 of of OpenStax *Calculus Volume 3*.) Show there is no plane perpendicular to $\vec{n} = \vec{i} + \vec{j}$ that passes through points $P(1, 2, 3)$ and $Q(2, 3, 4)$.

Problem 5. (Problem 386 from section 2.7 of of OpenStax *Calculus Volume 3*.)
Find the rectangular coordinates for the point with spherical coordinates $(1, \frac{\pi}{6}, \frac{\pi}{6})$.

Problem 6. (Problem 390 from section 2.7 of of OpenStax *Calculus Volume 3*.)
Find the spherical coordinates for the point with rectangular coordinates $(-1, 2, 1)$. You may use a calculator to evaluate inverse trigonometric functions encountered in this problem.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.

I will use the following guidelines in grading this problem set:

- What I expect (8 points). Your written solutions and verbal explanations show that you understand (1) how to create equations for lines in space, (2) how to create equations for planes in space, (3) how to use the equations for lines and planes reason about relationships between points, lines, and planes, and (4) the relationship between the spherical and rectangular coordinates for a point.
- Three quarters of what I expect (6 points). A plausible but not exclusive example is showing that you fully understand 4 of the expected items and partially or completely fail to understand the remaining one.
- Half of what I expect (4 points). Plausible but non-exclusive examples include showing that you understand 2 of the expected items, with no understanding of the others, OR showing that you partially but not completely understand all the expected items.
- Exceeding what I expect (typically 1 point added to what you otherwise earn). One plausible way of exceeding expectations that is specific to this problem set is

to produce high-quality graphs of the geometries of some of the problems. Demonstrating in other ways that you have significantly engaged with math beyond what is minimally needed to solve the given problems also exceeds what I expect.