Problem Set 2 — Vectors

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Math 223 04

Complete By Wednesday, January 31
Grade By Monday, February 5

Purpose

This problem set mainly develops your understanding of 3 dimensional vectors, although it also offers a little bit of practice with designing and plotting 3 dimensional surfaces.

Background

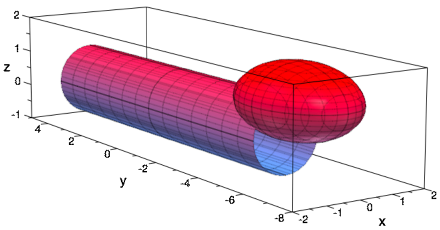
This problem set is based on material from the Sections 2.2 through 2.3 of our textbook. We will cover that material in class on January 25 and 26.

Problem 1 asks you to design some surfaces and plot them with muPad. We covered muPad in class of January 22, and surfaces on January 24. Sections 2.6 and parts of 2.2 in the textbook are also relevant.

Activity

Solve the following problems.

Problem 1. Remember the computer animated snake from the first day of class? While we haven't yet seen enough 3 dimensional math to generate very convincing snakes, we can generate slightly silly approximations. For example, here is a "snake" that consists of two surfaces. The body is a circular cylinder of radius 1 unit, aligned with the y axis, and plotted over the interval $-5 \leq y \leq 5$. The head is an ellipsoid with a radius of $\sqrt{2}$ units in the x direction, 1 unit in the z, and 2 units in the y, centered at $(0, -5.5, 1)$:



Find equations for the surfaces that make up this “snake.” Then use muPad or a similar tool to plot those equations to verify that they look like the picture. The easiest way to define the body is as an infinitely long cylinder, with the interval over which muPad plots it set to $-5 \leq y \leq 5$ in order to make it appear finite. This is fine.

Problem 2. (Inspired by problem 86 in section 2.2 of OpenStax *Calculus Volume 3*.)
Let $\vec{a}$ be the vector $2\vec{i} + \vec{j} - \vec{k}$ and $\vec{b}$ be the vector $\vec{i} - 2\vec{j} - 3\vec{k}$. Find $2\vec{a} - 5\vec{b}$ and express in component form.

Problem 3. (Problem 100 in section 2.2 of OpenStax *Calculus Volume 3*.)
Let $\vec{v} = \langle 2, 4, 1 \rangle$, and let $\vec{u}$ be a vector with magnitude 15 that points in the same direction as $\vec{v}$. Find $\vec{u}$.

Problem 4. Let $\vec{u} = \langle 1, 0, 3 \rangle$ and $\vec{v} = \langle -2, 4, 2 \rangle$. Find $\vec{u} \cdot \vec{v}$.

Problem 5. (Based on problem 170a in section 2.3 of OpenStax *Calculus Volume 3*.)
Let $\vec{u} = \langle 4, 4, 0 \rangle$ and $\vec{v} = \langle 0, 4, 1 \rangle$. Find the vector projection of $\vec{v}$ onto $\vec{u}$. Also find the angle, in degrees, between the vectors. You may use a calculator or mathematical software package to do the arithmetic in this problem, but set up that arithmetic by hand.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the “Grade By” date above.

I will use the following guidelines in grading this problem set:

- What I expect (8 points). Your written solutions and verbal explanations show that you understand (1) how to construct equations for the surfaces of the “snake,” (2) how to plot those equations as a single plot with muPad or a similar tool, (3) how to solve problems involving vector arithmetic and magnitude, (4) how to compute dot products, and (5) how to use the dot product in problem solving.

- Half of what I expect (4 points). Plausible but non-exclusive examples include showing that you understand 2 to 3 of the expected items, with no understanding of the others, OR showing that you partially but not completely understand all the expected items.

- Exceeding what I expect (typically 1 point added to what you otherwise earn). One plausible way of exceeding expectations that is specific to this problem set is to find ways to include other kinds of surface in the "snake." Some non-exclusive examples of more general ways include particularly elegant solutions OR particularly effectively coordinating the verbal explanation and written work OR avoiding all arithmetic, notational, and similar errors in your work OR extending the problems with additional work that deepens your understanding of the underlying math, etc.