Problem Set 1 — 3 Dimensional Coordinates

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Math 223 04

Complete By Tuesday, January 23
Grade By Friday, January 26

Purpose

This problem set develops your understanding of points, coordinates, and equations in 3 dimensional space.

Background

This problem set is based on material from the "Three Dimensional Coordinate Systems" and "Writing Equations in $\mathbb{R}^3$" subsections of section 2.2 of our textbook. We covered that material in class on January 19. Problem 4 also asks you to use muPad to graph equations in three dimensions, something we will cover in class on January 22.

Activity

Solve the following problems.

Problem 1. (Inspired by exercise 61 in section 2.2 of Openstax *Calculus Volume 3*.)

Imagine a rectangular box that has one corner at the origin and the diagonally opposite corner at point $(3, 1, 4)$. See exercise 61 (the first exercise in the section) in section 2.2 of our textbook for a diagram that illustrates the geometry, although it has different coordinates for the corner of the box.

Part 1. Give the coordinates of the other corners of the box.

Part 2. What is the length of the diagonal of the box?

Problem 2. Consider the points in 3 dimensional space that satisfy the constraints $-1 \leq x \leq 1$, $y \geq 0$, and $z = 3$.

Part 1. Describe the region that contains these points in English.

Part 2. For each of the following points, say whether or not it is in the region and why or why not:

- $(0, 3, 1)$

- $(0, 1, 3)$
- $(1, -1, 3)$

Part 3. Identify one point not suggested in Part 2 that is in the region.

Problem 3. Consider the equation $(x + 1)(y - 2)(z - 6) = 0$.

Part 1. This equation defines a surface (or set of intersecting surfaces). Describe that surface (or set of surfaces) in English.

Part 2. For each of the following points, say whether or not it lies on the surface(s) defined by the equation, and why or why not:

- $(0, 0, 0)$
- $(1, 2, 3)$
- $(-1, 0, 0)$

Part 3. Identify one point not suggested in Part 2 that lies on the surface(s).

Problem 4. Use muPad to plot the surface defined by the equation

$$x^2 + \frac{y^2}{4} + z^2 = 9$$

Pick a range of x, y, and z values to plot over that gives a reasonable sense of the shape of the surface.

Problem 5. Show that the set of points equidistant from $(3, 0, 0)$ and $(1, 0, 0)$ is the plane $x = 2$ (in other words, show that a point $P = (x, y, z)$ is the same distance from $(3, 0, 0)$ as it is from $(1, 0, 0)$ if and only if P lies in the $x = 2$ plane).

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.

I will use the following guidelines in grading this problem set:

- What I expect (8 points). Your written solutions and verbal explanations show that you understand (1) the use of coordinate triples to define points in space, (2) how to find distances between points in space, (3) how equalities and inequalities can correspond to regions in space, (4) how the solutions to equations can correspond to surfaces in space, and (5) how to graph 3-dimensional equations with muPad. I expect you to use the concept of distance in a relevant way in problem 5, but not to write a completely formal proof.

- Half of what I expect (4 points). Plausible but non-exclusive examples include showing that you understand 2 to 3 of the expected items, with no understanding of the others, OR showing that you partially but not completely understand all the expected items.

- Exceeding what I expect (typically 1 point added to what you otherwise earn). Plausible but non-exclusive examples include particularly elegant solutions OR particularly effectively coordinating the verbal explanation and written work OR avoiding all arithmetic, notational, and similar errors in your work OR extending the problems with additional work that deepens your understanding of 3 dimensional coordinate systems OR having a formal proof for problem 5, etc.