Problem Set 9 — Extreme Values, Optimization, and Multiple Integrals

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Math 223 01

Complete By Tuesday, November 6
Grade By Thursday, November 8

Purpose

This problem set reinforces your understanding of several topics we have covered recently: extreme values and optimization for multivariable functions, and integrals of such functions.

Background

Section 4.7 of our textbook covers finding and classifying critical points We discussed the parts of that section relevant to this problem set in class on October 24.

Optimization of multivariable functions is covered in section 4.8, which we discussed in class on October 26 and 31.

Integration of multivariable functions as exercised in this problem set is covered in sections 5.1, 5.2, and 5.4, which we discussed in class on November 1 and 2.

Activity

Solve the following problems. Remember not to use calculators or computers except when explicitly told you may.

Problem 1. (Exercise 318 in section 4.7 of OpenStax *Calculus Volume 3.*)

Find the critical points of $f(x, y) = -x^3+4xy-2y^2+1$, and use the second derivative test to determine which (if any) are minima, maxima, and saddle points.

In addition, use Mathematica to plot the function around the critical points you found to check your results visually.

Problem 2. (Based on exercise 370 in section 4.8 of OpenStax *Calculus Volume 3.*)

Find the values of x and y that minimize and maximize the product xy if point (x, y) must lie on the circle $x^2 + y^2 = 1$.

Problem 3. Evaluate

$$\int_0^1 \int_1^2 \int_0^2 xyz \, dz \, dy \, dx$$

Check your answer by evaluating the integral with Mathematica.

Problem 4. (Exercise 100 in section 5.2 of OpenStax *Calculus Volume 3.*)
Evaluate

$$\iint_D (x^2 + y) \, dA$$

where region D is the region between the curves $y = -4 + x^2$ and $y = 4 - x^2$. See the book for a sketch of the region.

Check your answer by evaluating the integral with Mathematica.

Problem 5. One of the properties of double integrals that our textbook states is that over a rectangular region $R = [a, b] \times [c, d]$

$$\iint_R g(x)h(y) \, dA = \left(\int_a^b g(x) \, dx \right) \left(\int_c^d h(y) \, dy \right)$$

Justify this claim by showing how to express the double integral from the left side of the equation as an iterated integral and then rearrange it into the product on the right side.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you work in a group on this problem set, all members of the group can come to the same meeting.

I will use the following guidelines in grading this problem set:

- What I expect (8 points). Your written solutions and verbal explanations show that you understand (1) how to find critical points of 2-variable functions, (2) how to identify critical points as corresponding to minima, maxima, or saddle points, (3) how to solve multivariable optimization problems, (4) how to evaluate multiple integrals over rectangular regions, (5) how to evaluate multiple integrals over non-rectangular regions, (6) how to evaluate integrals of multivariable functions with Mathematica, and (7) how to reason abstractly about properties of multiple integrals.

- Half of what I expect (4 points). Plausible but non-exclusive examples include failing to understand 3 or 4 of the expected items and understanding the others, OR showing that you partially but not completely understand all the expected items.

- Exceeding what I expect (typically 1 point added to what you otherwise earn). Generally, demonstrating that you have nontrivially engaged with math in ways beyond what is needed to solve the given problems exceeds my expectations. One, but not the only, way to do this on this problem set would be to find ways to use Mathematica to check your answers beyond what I already ask you to do.