# Problem Set 8 — Advanced Multivariable Derivatives

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Math 223 01

**Complete By** Tuesday, October 30
**Grade By** Thursday, November 1

## Purpose

This problem set reinforces your understanding of derivatives of multivariable functions beyond the definition and basic laws of such derivatives. In particular, it develops your understanding of the chain rule for partial derivatives, and of gradients and directional derivatives.

## Background

This exercise is based on sections 4.5 and 4.6 of our text, which we covered in class between October 18 and 22.

## Activity

Solve the following problems. Remember not to use calculators or computers except when explicitly told you may.

**Problem 1.** Suppose $f(x, y, z)$ is a differentiable function of three variables, and that those three variables represent the coordinates of points along some (also differentiable) curve $\vec{r}(t)$. You can thus say that $f(x, y, z) = f(\vec{r}(t))$. Show that

$$\frac{df}{dt} = \nabla f \cdot \vec{r'}(t)$$

where the gradient is with respect to $x$, $y$, and $z$.

**Problem 2.** (Exercise 252 in section 4.5 of OpenStax *Calculus Volume 3*.)
Starting with the equation

$$PV = kT$$

that relates the pressure ($P$), volume ($V$) and temperature ($T$) of a gas, find $\frac{dV}{dt}$ given information about $P$, $T$, and their derivatives. See the textbook for the details.

**Problem 3.** (Exercise 272 in section 4.6 of OpenStax *Calculus Volume 3*.)
Find the directional derivative at $(5, 5)$ of $f(x, y) = x^2y$ in direction $\langle 3, -4\rangle$.

**Problem 4.** Let $z = x^2 + y^2$. Show that no matter what direction you move from $(x, y) = (0, 0)$, the instantaneous rate of change in $z$ is 0.

**Problem 5.** Find the gradient of $f(x, y, z) = \ln(x^2y + y^3z + zx)$ in terms of $x$, $y$, and $z$. Then find the numeric value of the gradient at point $(1, 2, 3)$.

**Problem 6.** (Exercise 306 in section 4.6 of OpenStax *Calculus Volume 3*.)
The temperature in a metal sphere is inversely proportional to distance from the origin. Show that at any point in the sphere the direction of greatest increase in temperature is towards the origin, and then find the rate of change in temperature at point $(1, 2, 2)$ in the direction towards point $(2, 1, 3)$. (See the textbook for a different wording of this problem.)

## Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you work in a group on this problem set, all members of the group can come to the same meeting.

I will use the following guidelines in grading this problem set:

- What I expect (8 points). Your written solutions and verbal explanations show that you understand (1) how to use the chain rule in problem solving, (2) how to calculate directional derivatives, (3) how to use directional derivatives in problem solving, (4) how to calculate gradients, and (5) how to use gradients in problem solving.

- Half of what I expect (4 points). Plausible but non-exclusive examples include failing to understand 2 or 3 of the expected items and understanding the others, OR showing that you partially but not completely understand all the expected items.

- Exceeding what I expect (typically 1 point added to what you otherwise earn). Generally, demonstrating that you have nontrivially engaged with math in ways beyond what is needed to solve the given problems exceeds my expectations. One, but not the only, way to do this on this problem set would be to find ways to use Mathematica to check your answers to some of the problems.