Problem Set 7 — Multivariable Derivatives

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Math 223 01

Complete By Sunday, October 21
Grade By Tuesday, October 23

Purpose

This problem set reinforces your ability to find and solve problems involving derivatives of multivariable functions.

Background

This exercise is based on sections 4.3 and 4.4 of our text, which we covered in class between October 12 and 17.

Activity

Solve the following problems. Remember not to use calculators or computers except when explicitly told you may.

Problem 1. Find all the first and second partial derivatives of

$$f(x, y, z) = xy^2z + x \cos z - z \ln(x + y)$$

Based on your results, how do you think Clairaut's Theorem (the mixed-derivative theorem) applies to functions of 3 variables?

Problem 2. Suppose $g(x, y) = xe^y$. Use Mathematica to find $\frac{\partial g}{\partial x}$ and $\frac{\partial^2 g}{\partial x \partial y}$.

Problem 3. Geneseo Widget Works, makers of fine widgets, employs managers and assemblers. The number of widgets per hour that the Widget Works can make when it has m managers and a assemblers is given by $w = \frac{ma}{10} - m$. Suppose the Widget Works has 2 managers and 20 assemblers, but wants to hire more workers. Will output rise fastest if the Widget Works hires assemblers, or if they hire managers (assume the only choice is to hire one kind of worker or the other, but not a combination)?

Problem 4. (Inspired by exercise 198 in section 4.4 of OpenStax *Calculus Volume 3*.) Calculate the value of $z = \sqrt{4 - x^2 - y^2}$ at $(x, y) = (1, 1)$. Then use a linear approximation to estimate the value at $(1.01, 0.97)$. Finally, calculate the actual value at $(1.01, 0.97)$ and compare it to your estimate. You may use a calculator for the numeric calculations in this problem.

Problem 5. The following table gives values for function $f(x, y)$ for certain values of x and y:

	$x = 1$	$x = 2$	$x = 3$	$x = 4$
$y = 1$	1.5	2	2.5	3
$y = 2$	2.5	3	3.5	4
$y = 3$	3.5	4	4.5	5
$y = 4$	4.5	5	5.5	6

Estimate the value of $f(2.1, 3.05)$.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you work in a group on this problem set, all members of the group can come to the same meeting.

I will use the following guidelines in grading this problem set:

- What I expect (8 points). Your written solutions and verbal explanations show that you understand (1) how to find first and second partial derivatives, (2) how to use Mathematica to find first and second partial derivatives, (3) how to use partial derivatives to estimate rates of change in multivariable functions, and (4) how to use linear approximations to estimate the value of a multivariable function.

- Half of what I expect (4 points). Plausible but non-exclusive examples include failing to understand 2 of the expected items and understanding the others, OR showing that you partially but not completely understand all the expected items.

- Exceeding what I expect (typically 1 point added to what you otherwise earn). Generally, demonstrating that you have nontrivially engaged with math in ways beyond what is needed to solve the given problems exceeds my expectations.