Problem Set 6 — Multivariable Functions

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Math 223 01

Complete By Monday, October 15
Grade By Wednesday, October 17

Purpose

This problem set reinforces your understanding of multivariable functions and their limits and derivatives.

Background

This exercise is based on sections 4.1 through 4.3 of our text, which we covered (or will cover) in class between October 10 and 15.

Some problems ask you to generate plots with Mathematica. We covered the relevant plotting commands in classes on October 10 and 11.

Activity

Solve the following problems. Remember not to use calculators or computers except when explicitly told you may.

Problem 1. (Based on exercise 14 in section 4.1 of OpenStax *Calculus Volume 3*.)

Give equations for, and use Mathematica to plot, the level curves for $c = 1$, $c = 2$, and $c = 3$ of the function $z(x, y) = y^2 - x^2$.

Problem 2. (Exercise 76 in section 4.2 of OpenStax *Calculus Volume 3*.)
Determine whether

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}$$

exists, and if so what it is.

Problem 3. (Inspired by exercise 86 in section 4.2 of OpenStax *Calculus Volume 3*.)

Part 1. Show that

$$\lim_{(x,y)\to(0,0)} \frac{xy + y^3}{x^2 + y^2}$$

does not exist.

Part 2. Use Mathematica to plot the function from part 1 near the origin. Be prepared during grading to identify the feature(s) of the plot that correspond to the non-existence of the limit.

Problem 4. Intuitively, a single-variable function, $f(x)$, is said to have a limit of $+\infty$ at a if you can make $f(x)$ arbitrarily large by picking values for x sufficiently close to a. Formally, the idea is that for every number M, there exists a positive number δ such that $f(x) > M$ whenever $a - \delta < x < a + \delta$.

Extend the intuitive and formal definitions of a (positive) infinite limit to multivariable functions. Then use your extended formal definition to show that

$$\lim_{(x,y)\to(0,0)} \frac{1}{x^2 + y^2} = \infty$$

Problem 5. Find all the (first) partial derivatives of each of the following functions:

1. $f(x, y) = \sin(xy) - xy$
2. $g(x, y) = x \ln(x + y)$
3. $h(x, y, z, w) = \frac{x^2-y^2}{z^2+w^2}$

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you work in a group on this problem set, all members of the group can come to the same meeting.

I will use the following guidelines in grading this problem set:

- What I expect (8 points). Your written solutions and verbal explanations show that you understand (1) what level curves are, (2) how to plot level curves with Mathematica, (3) how to determine whether a limit of a multivariable function exists, (4) how to find limits of multivariable functions when those limits exist, (5) how to plot multivariable functions with Mathematica, and (6) how to evaluate partial derivatives. I also expect that (7) you will be able to take some steps towards defining an infinite multivariable limit and applying that definition (problem 4), although maybe not completely solve the problem.

- Three quarters of what I expect (6 points). Plausible, but not exclusive, examples include failing to understand 1 or 2 of the expected items, OR having errors beyond arithmetic or typographical mistakes in one or more solutions even though you generally understand the expected items.

- Half of what I expect (4 points). Plausible but non-exclusive examples include failing to understand 3 to 5 of the expected items, OR showing that you partially but not completely understand all the expected items.
- Exceeding what I expect (typically 1 point added to what you otherwise earn). Generally, demonstrating that you have nontrivially engaged with math in ways beyond what is needed to solve the given problems exceeds my expectations. A natural (but not exclusive) way this could happen on this problem set would be giving a complete definition and application of infinite multivariable limits (problem 4).