Problem Set 5 — Arc Length and Curvature

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Math 223 01

Complete By Wednesday, October 3
Grade By Friday, October 5

Purpose

This problem set reinforces your understanding of arc length and curvature of vector valued functions.

Background

This exercise is based on section 3.3 of our text, which we covered (or will cover) in class between September 27 and October 3.

One problem asks you to numerically evaluate an integral with Mathematica, a subject we covered in class on September 28.

Activity

Solve the following problems. Remember not to use calculators or computers except when explicitly told you may.

Problem 1. (Inspired by exercise 107 in section 3.3 of OpenStax *Calculus Volume 3.*)

Find the length of one turn of the 4-dimensional helix $\vec{r}(t) = \langle 2\sin t, \sqrt{3}t, 2\cos t, 2t\rangle$.

(Be prepared when grading this problem to discuss how you came up with the formula(s) you used for the calculation and whether it even makes sense to talk about a length in 4 dimensions.)

Problem 2. (Inspired by exercise 110 in section 3.3 of OpenStax *Calculus Volume 3.*)

Set up an integral to find the circumference of the ellipse $\vec{r}(t) = \langle \cos t, 2\sin t\rangle$. Use Mathematica to numerically evaluate this integral.

Problem 3. An ant crawls along the curve $\vec{r}(t) = \langle 2t^2, t^2 - 1, \frac{\sqrt{5}}{2}t^2\rangle$, starting at $\vec{r}(1)$. The ant moves in the direction of increasing t, i.e., it moves through points on the curve associated with ever larger t values. Find the coordinates of the point the ant is at after it has crawled a distance of 1 unit.

Problem 4. Find a unit vector that points in the direction the curve $\vec{r}(t) = \langle \cos(e^t), \sin(e^t), 0 \rangle$ is turning when $t = \ln \pi$.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you work in a group on this problem set, all members of the group can come to the same meeting.

I will use the following guidelines in grading this problem set:

- What I expect (8 points). Your written solutions and verbal explanations show that you understand (1) how to find arc lengths, (2) how to evaluate integrals with Mathematica, (3) how to work backwards from arc lengths to corresponding points on a curve, and (4) how to find the direction in which a parametric curve turns.
- Three quarters of what I expect (6 points). Plausible, but not exclusive, examples include failing to understand 1 of the expected items, OR having errors beyond arithmetic or typographical mistakes in one or more solutions even though you generally understand the expected items.
- Half of what I expect (4 points). Plausible but non-exclusive examples include failing to understand 2 of the expected items, OR showing that you partially but not completely understand all the expected items.
- Exceeding what I expect (typically 1 point added to what you otherwise earn). Generally, demonstrating that you have nontrivially engaged with math in ways beyond what is needed to solve the given problems exceeds my expectations. Natural (but not exclusive) ways this could happen on this problem set include exploring Mathematica beyond what I ask or trying multiple ways of solving/checking solutions to some of the problems.