Problem Set 4 — Calculus of Vector Valued Functions

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Math 223 01

Complete By Thursday, September 27
Grade By Monday, October 1

Purpose

This problem set reinforces your understanding of vector valued functions and their limits, derivatives, and integrals.

Background

This exercise is based on sections 3.1 and 3.2 of our text, which we covered (or will cover) in class between September 19 and 24.

Some problems ask you to plot vector valued functions with Mathematica, a subject we covered in class on September 19.

Activity

Solve the following problems.

Problem 1. Let $\vec{r}(t) = \langle \frac{1-\sin^2 t}{\cos t}, \frac{t^2+t-2}{t-1}, \tan \frac{t}{2} \rangle$. Find each of the following limits, if they exist:

$$\lim_{t \to \frac{\pi}{2}} \vec{r}(t)$$

$$\lim_{t \to 1} \vec{r}(t)$$

$$\lim_{t \to \pi} \vec{r}(t)$$

Problem 2. Let $\vec{r}(t) = \langle t^2 + 3t, e^t \cos(e^t), \sqrt{t+1} \rangle$.

Part 1. Use Mathematica to plot $\vec{r}(t)$ between $t = -1$ and $t = 1$. (Note: Mathematica has a built-in function, Exp[t], that calculates e^t.)

Part 2. Find the derivative of $\vec{r}(t)$ with respect to t.

Part 3. Find the general antiderivative of $\vec{r}(t)$ with respect to t.

Problem 3. An ant is crawling along a coil of wire in such a manner that t seconds after the ant starts crawling it is at position $\langle 2t, \sin t, \cos t\rangle$, in some coordinate system in which distance is measured in inches.

Part 1. Use Mathematica to plot the ant's position between $t = 0$ and $t = 3$.

Part 2. Find a function for the ant's velocity as a function of time.

Part 3. Find a function for the ant's speed as a function of time. (Recall the relationship between "velocity" and "speed.")

Part 4. Calculate the ant's velocity and speed 2π seconds after it starts crawling.

Problem 4. An object starts at rest at point $(1, 2, 0)$ and accelerates with acceleration $\vec{a}(t) = \langle 0, 1, 2\rangle$ (with magnitude measured in meters per second per second). Find the location of the object after 2 seconds.

Problem 5. Since vector valued functions produce vectors, you can also compute the dot product of two vector valued functions.

Part 1. Find the dot product $\vec{f}(t) \cdot \vec{g}(t)$ where $\vec{f}(t) = \langle t \sin t, t, 3t^2\rangle$ and $\vec{g}(t) = \langle \sin t, \cos^2 t, \frac{1}{t}\rangle$.

Part 2. Show that for any 3-dimensional vector valued functions $\vec{f}(t)$ and $\vec{g}(t)$ and constant c, such that $\lim_{t\to c} \vec{f}(t)$ and $\lim_{t\to c} \vec{g}(t)$ exist, $\lim_{t\to c}(\vec{f}(t) \cdot \vec{g}(t)) = (\lim_{t\to c} \vec{f}(t))\cdot(\lim_{t\to c} \vec{g}(t))$.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you work in a group on this problem set, all members of the group can come to the same meeting.

I will use the following guidelines in grading this problem set:

- What I expect (8 points). Your written solutions and verbal explanations show that you understand (1) how to plot vector valued functions with Mathematica, (2) how to find limits of vector valued functions, (3) how to differentiate vector valued functions, (4) how to integrate vector valued functions, (5) how to use derivatives of vector valued functions to solve problems, and (6) how to use integrals of vector valued functions to solve problems.

- Three quarters of what I expect (6 points). Plausible, but not exclusive, examples include failing to understand 1 or 2 of the expected items, OR having errors beyond arithmetic or typographical mistakes in one or more solutions even though you generally understand the expected items.

- Half of what I expect (4 points). Plausible but non-exclusive examples include failing to understand 3 or 4 of the expected items, OR showing that you partially but not completely understand all the expected items.

- Exceeding what I expect (typically 1 point added to what you otherwise earn). Generally, demonstrating that you have nontrivially engaged with math in ways beyond what is needed to solve the given problems exceeds my expectations. Some natural (but not exclusive) ways this could happen on this problem set include exploring Mathematica beyond what I ask, or generalizing your solutions beyond what I ask.