Problem Set 3 — Lines and Planes

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Math 223 01

Complete By Tuesday, September 18
Grade By Thursday, September 20

Purpose

This problem set reinforces your understanding of lines and planes in space. In the process, it also consolidates your ability to work with vectors.

Background

This exercise is mostly based on section 2.5 of our text, which we covered (or will cover) in class on September 13 and 14. Some of the exercise also draws on material in sections 2.3 and 2.4, covered on September 10 and 12, respectively.

Activity

Solve the following problems.

Problem 1. (Problem 190 in section 2.4 of OpenStax *Calculus Volume 3*.)

Find a unit vector that points in the same direction as the cross product of $\langle 2, 6, 1\rangle$ and $\langle 3, 0, 1\rangle$. Express your answer in either component form or in terms of standard unit vectors, as you prefer.

Problem 2. (Part of problem 221a in section 2.4 of OpenStax *Calculus Volume 3*.)

Prove that if $\vec{u}$ and $\vec{v}$ are 3-dimensional vectors, and c is a numeric constant, then

$$c(\vec{u} \times \vec{v}) = (c\vec{u}) \times \vec{v}$$

Problem 3. (Problem 248 from section 2.5 of OpenStax *Calculus Volume 3*.)

Find parametric and symmetric equations for the line through point $(3, 1, 5)$ in direction $\langle 1, 1, 1\rangle$. Also find the point at which this line intersects the xy plane.

Problem 4. (Based on problem 268 from section 2.5 of OpenStax *Calculus Volume 3*.)

Find an equation for the plane that contains point $(3, 2, 2)$ and has normal vector $\langle 2, 3, -1\rangle$.

Problem 5. Part 1. Given a line, $\vec{r}(t) = P + t\vec{v}$ and a plane $ax + by + cz = d$, explain a test you could use to determine whether the line is parallel to the plane.

Part 2. Use your test to determine whether the following lines are parallel to plane $x + y + z = 1$:

1. $\vec{r}(t) = (1, 0, 0) + t\langle 2, 1, 0\rangle$
2. $\vec{r}(t) = (2, 1, 2) + t\langle 1, 0, -1\rangle$

Part 3. (Problem 276 from section 2.5 of OpenStax *Calculus Volume 3.*) Show there is no plane perpendicular to $\vec{n} = \vec{i} + \vec{j}$ that passes through points $P(1, 2, 3)$ and $Q(2, 3, 4)$.

Problem 6. Consider the plane $x + 2y - z = 10$ and the line $(1, 0, 0) + t\langle 1, 1, 0\rangle$.

Part 1. Find the coordinates of the point where the line intersects the plane.

Part 2. Find the angle between the line and the plane.

Do not use a calculator in solving this problem.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you work in a group on this problem set, all members of the group can come to the same meeting.

I will use the following guidelines in grading this problem set:

- What I expect (8 points). Your written solutions and verbal explanations show that you understand (1) how to compute cross products, (2) what other vector operations are needed to solve the given problems and how to carry them out, (3) how to prove properties of cross products, (4) how to find equations for lines, (5) how to find equations for planes, and (6) how to use equations for lines and planes to reason about their interactions.
- Three quarters of what I expect (6 points). Plausible, but not exclusive, examples include failing to understand 1 or 2 of the expected items, OR having errors beyond arithmetic or typographical mistakes in one or more solutions even though you generally understand the expected items.
- Half of what I expect (4 points). Plausible but non-exclusive examples include failing to understand 3 or 4 of the expected items, OR showing that you partially but not completely understand all the expected items.
- Exceeding what I expect (typically 1 point added to what you otherwise earn). Generally, demonstrating that you have nontrivially engaged with math in ways beyond what is needed to solve the given problems exceeds my expectations. This could include, but isn't limited to, finding ways to test your solutions to problems, extending what you do with a problem beyond what I ask, etc.