Problem Set 2 — Vectors

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Math 223 01

Complete By Wednesday, September 12
Grade By Friday, September 14

Purpose

This problem set mainly ensures your familiarity with the basics of three-dimensional vectors. One of the problems also helps reinforce your understanding of cylindrical coordinates.

Background

This exercise is based on the second half (roughly) of section 2.2, and all of sections 2.3 and 2.7, in our textbook. We talked about this material in class between September 6 and 10.

Activity

Solve the following problems.

Problem 1. The equation $z = x^2 + y^2$ defines a paraboloid in rectangular coordinates. What is the equation of that same paraboloid in cylindrical coordinates?

Problem 2. (Inspired by problem 86 in section 2.2 of OpenStax *Calculus Volume 3*.)

Let $\vec{a}$ be the vector $2\vec{i} + \vec{j} - \vec{k}$ and $\vec{b}$ be the vector $\vec{i} - 2\vec{j} - 3\vec{k}$. Find $2\vec{a} - 5\vec{b}$ and express in component form.

Problem 3. An airplane is flying in such a manner that its velocity relative to the surrounding air is given by the vector $\langle 100, 20, 5 \rangle$. The air is moving with respect to the surface of the earth with velocity $\langle 20, -10, 0 \rangle$, and the surface of the earth is moving with respect to a hypothetical fixed point at the center of the earth with a velocity of $\langle 1000, 0, 0 \rangle$. (If you care, I imagine all of these vectors defined in a coordinate system in which the x axis points east, the y axis points north, and the z axis points up, but that doesn't matter to solving the problem.)

Part 1. What is the airplane's velocity with respect to the fixed point at the center of the earth?

Part 2. What is the airplane's speed with respect to that point? You may use a calculator to do the arithmetic for this problem, but set up the arithmetic by hand. It may help to recall that speed is the magnitude of velocity.

Problem 4. Prove that scalar multiplication distributes over vector addition for three-dimensional vectors. In other words, prove that if $\vec{u}$ and $\vec{v}$ are vectors and c is a real number, $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$.

As a way of going beyond what I expect for this problem, prove this for any number of dimensions, not just three.

Problem 5. Let $\vec{u} = \langle 1, 0, 3 \rangle$ and $\vec{v} = \langle -2, 4, 2 \rangle$. Find $\vec{u} \cdot \vec{v}$.

Problem 6. (Inspired by problem 173 in section 2.3 of OpenStax *Calculus Volume 3*.)

Water molecules consist of an oxygen atom and two hydrogen atoms. Suppose the oxygen atom in a certain water molecule is centered at the origin, and the hydrogen atoms are centered at points $(-1, 1, -1)$ and $(-1.06, -1.06, 0.87)$. What is the angle between the two oxygen-hydrogen bonds in this molecule? (Assume that the bonds lie along the lines connecting the centers of each hydrogen atom to the center of the oxygen atom.)

You may use a calculator to do the calculations for this problem, but set up those calculations by hand.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you work in a group on this problem set, all members of the group can come to the same meeting.

I will use the following guidelines in grading this problem set:

- What I expect (8 points). Your written solutions and verbal explanations show that you understand (1) how to convert equations in rectangular coordinates to cylindrical coordinates, (2) how to compute sums, differences, and scalar products of vectors, (3) how to compute dot products of vectors, (4) how to use vectors and vector arithmetic to model physical situations, and (5) how to reason with the definitions of vector addition and scalar product.

- Three quarters of what I expect (6 points). Plausible, but not exclusive, examples include failing to understand 1 of the expected items, OR having errors beyond arithmetic or typographical mistakes in one or more solutions even though you generally understand the expected items.

- Half of what I expect (4 points). Plausible but non-exclusive examples include failing to understand 2 or 3 of the expected items, OR showing that you partially but not completely understand all the expected items.

- Exceeding what I expect (typically 1 point added to what you otherwise earn). Generally, demonstrating that you have nontrivially engaged with math in ways beyond what is needed to solve the given problems exceeds my expectations. See the note in problem 4 for a specific way of exceeding my expectations for it.