Problem Set 11 — Scalar Line Integrals

Prof. Doug Baldwin

Math 223 01

Complete By Sunday, November 26
Grade By Tuesday, November 28

Purpose

This problem set mainly reinforces your understanding of scalar line integrals and their applications. As secondary goals, it also gives you a little more practice with multiple integrals for moment calculations, and introduces you to vector fields. Specific things I expect you to be able to do by the time you finish this problem set include ...

- Set up integrals for moments in 3 dimensions
- Evaluate multivariable integrals with Mathematica
- Evaluate scalar line integrals
- Understand how a vector field maps points to vector values
- Plot vector fields with Mathematica.

Background

Scalar line integrals are discussed at the beginning of section 6.2 in our textbook. We talked about them in class on November 14.

Moments are in section 5.6 of the book, discussed on November 9.

The material on vector fields for this problem set comes from the beginning of section 6.1, which I plan to discuss in class on November 15.

Activity

Solve the following problems. Remember not to use calculators or computers except when explicitly told you may.

Problem 1. Use Mathematica to solve exercise 338 in section 5.6 of OpenStax *Calculus Volume 3* (calculate the mass, moments about each plane, and center of mass of a solid occupying the region $0 \leq x \leq 1$, $0 \leq y \leq 2$, $0 \leq z \leq 3$ and with density $\rho(x, y, z) = x + y + 1$).

Problem 2. (Based on exercise 44 in section 6.2 of OpenStax *Calculus Volume 3*.)
Evaluate by hand:

$$\int_C (x + y)\, ds$$

where C is the path defined by $\langle t, 1 - t, 0\rangle$ from point $(0, 1, 0)$ to point $(1, 0, 0)$.

Problem 3. (Based on exercise 47 in section 6.2 of OpenStax *Calculus Volume 3*.)
Evaluate by hand:

$$\int_C xy^4\, ds$$

where C is the right half of the circle $x^2 + y^2 = 16$ and is traversed clockwise.

Problem 4. Evaluate by hand

$$\int_C x + \sqrt{y} - z^2\, ds$$

where C is the 2-part path that follows a straight line from point $(0, 0, 0)$ to point $(1, 0, 0)$, and then from there to point $(1, 1, 1)$.

Problem 5. Consider the vector field

$$\vec{F}(x, y) = \langle \frac{-x}{x^2 + y^2 + 1}, \frac{-y}{x^2 + y^2 + 1} \rangle$$

Part 1. What is the value of this field at point $(1, 2)$? How about at points $(0, 0)$ and $(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2})$?

Part 2. Use Mathematica to plot this field over the region $-3 \leq x \leq 3$, $-3 \leq y \leq 3$.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you work in a group on this problem set, all members of the group can come to the same meeting.

I will use the guidelines discussed in class on November 12 to grade this problem set.