Problem Set 1 — Three Dimensional Coordinate Systems

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Math 223 01

Complete By Thursday, September 6
Grade By Monday, September 10

Purpose

This problem set reinforces your understanding of 3-dimensional coordinate systems and some common surfaces in them.

Background

This exercise is based on the first half (roughly) of section 2.2, and all of section 2.6, in our textbook. We talked about, or will talk about, this material in class between August 29 and September 5.

This exercise also expects you to draw 3-dimensional graphs with Mathematica. This isn't in our textbook, but we discussed it in class on August 31.

Activity

Solve the following problems.

Problem 1. (Inspired by exercise 61 in section 2.2 of Openstax *Calculus Volume 3*.)

Imagine a rectangular box that has one corner at the origin and the diagonally opposite corner at point $(3, 1, 4)$. See exercise 61 in section 2.2 of our textbook for a diagram that illustrates the geometry, although it has different coordinates for the corner of the box.

Part 1. Give the coordinates of the other corners of the box.

Part 2. What is the length of the diagonal of the box?

Problem 2. Consider the points in 3 dimensional space that satisfy the constraints $-1 \leq x \leq 1$, $y \geq 0$, and $z = 3$.

Part 1. Describe the region that contains these points in English.

Part 2. For each of the following points, say whether or not it is in the region and why or why not:

- $(0, 3, 1)$
- $(0, 1, 3)$
- $(1, -1, 3)$

Part 3. Identify one point not listed in Part 2 that is in the region.

Problem 3. Consider the equation $(x + 1)(y - 2)(z - 6) = 0$.

Part 1. This equation defines a surface (or set of intersecting surfaces). Describe that surface (or set of surfaces) in English.

Part 2. For each of the following points, say whether or not it lies on the surface(s) defined by the equation, and why or why not:

- $(0, 0, 0)$
- $(1, 2, 3)$
- $(-1, 0, 0)$

Part 3. Identify one point not listed in Part 2 that lies on the surface(s).

Problem 4. Identify in English the kind of quadric produced by the equation $y^2 + z^2 - x^2 = 4$. Plot this equation with Mathematica to check your answer.

Problem 5. Part 1. Give an equation for an elliptic paraboloid with the following features:

- The paraboloid's axis is along the line $x = 4, y = 0$
- The paraboloid opens as you move in the positive z direction along its axis
- The paraboloid has a circular cross section
- The paraboloid's trace in the $z = 1$ plane has radius 1.

Part 2. Plot your function with Mathematica and verify that it looks like the description in Part 1.

Problem 6. Show that the set of points equidistant from $(3, 0, 0)$ and $(1, 0, 0)$ is the plane $x = 2$ (in other words, show that a point $P = (x, y, z)$ is the same distance from $(3, 0, 0)$ as it is from $(1, 0, 0)$ if and only if P lies in the $x = 2$ plane).

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along. This "written solution" should include either a printout of or a computer containing the Mathematica code you used in problems 4 and 5.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you work in a group on this problem set, all members of the group can come to the same meeting.

I will use the following guidelines in grading this problem set:

- What I expect (8 points). Your written solutions and verbal explanations show that you understand (1) how to describe and identify points, regions, and surfaces in space, (2) how to identify and define common quadric surfaces, (3) how to plot surfaces with Mathematica. I further expect (4) that you have some useful ideas about the proof in problem 6, although perhaps not a complete formal proof.

- Three quarters of what I expect (6 points). Plausible, but not exclusive, examples include failing to understand 1 of the expected items, OR having errors beyond arithmetic or typographical mistakes in one or more solutions even though you generally understand the expected items.

- Half of what I expect (4 points). Plausible but non-exclusive examples include showing that you understand 1 or 2 of the expected items, with no understanding of the others, OR showing that you partially but not completely understand all the expected items.

- Exceeding what I expect (typically 1 point added to what you otherwise earn). Generally, demonstrating that you have nontrivially engaged with math in ways beyond what is needed to solve the given problems exceeds my expectations. For this problem set, having a complete and correct proof for problem 6 would also exceed expectations.