# Math 223 — Hour Exam 1

February 26, 2018

**General Directions.** This is an open-book, open-notes, open-computer test. However, you may not communicate with any person, except me, during the test. You have the full class period (50 minutes) in which to do the test. Put your answer to each question in the space provided (use the backs of pages if you need more space). Be sure to **show** **your** **work**! I give partial credit for incorrect answers if you show correct steps leading up to them; conversely, I do not give full credit even for correct answers if it is not clear that you understand where those answers come from. Good luck.

This test contains 5 questions on 4 pages.

**Question 1** (5 Points). An airplane is moving with constant velocity, in such a manner that every second its change in position is given by the vector feet. Find the vector that gives the change in the plane’s position over a period of 5 seconds.

**Solution.** Since the velocity doesn’t vary with time, you can just multiply by 5 to find the change in position over 5 seconds:

**Question 2** (15 Points). Suppose a particle with initial velocity accelerates with acceleration

meters per second per second, where *t* represents time in seconds. Is there ever a time when the particle is standing still, i.e., has velocity ? If so, when? If not, why not?

**Solution.** First, find velocity as the indefinite integral of acceleration, using the initial velocity to find the constant of integration:

Now see if it is possible for all three components of the velocity to simultaneously be 0:

but…

So since ½ and √2 are different numbers, there is no *t* value that makes even the first two components 0, let alone all three.

**Question 3** (5 Points). Consider the cylinder defined by lines that are parallel to vector and that pass through the curve . Find one point that lies on this cylinder but not in the *z* = 0 plane.

**Solution.** Points on the cylinder are all of the form (0,*y*,*y*2)+ *t*⟨2,1,0⟩ for arbitrary choices of *y* and *t*. So pick any *y* and *t* that are easy to work with and don’t yield *z* (i.e., *y*2) = 0, for example *y* = 1 and *t* = 0. This combination yields the point (0,1,1).

**Question 4** (10 Points). A dust mote in a tornado follows the path

Find a vector that describes the instantaneous direction in which the mote is moving when *t* = 0.

**Solution.** The instantaneous direction is the direction the derivative of points, so a minimal answer is simply :

A slightly more elegant answer might be a unit vector pointing in the same direction as :

**Question 5** (15 Points). Suppose I want to find one or more planes perpendicular to vector such that each of the points (0,0,0), (-2,0,3), and (2,1,-4) lies in one of the planes — several points may lie in the same plane, or each point may lie in its own distinct plane, as long as for each point there is some plane to contain it. What is the minimum number of planes I need, and what are their equations?

**Solution**. Plane equations can be written in the form

where *A*, *B*, and *C* are the components of the plane’s normal: 1, 2, and 1 respectively, in this case. So find the *D* values for each of the given points; any points that produce the same *D* lie in the same plane, and conversely the minimum number of planes needed is the number of distinct *D* values:

So I need two planes, with equations