# Math 223 — Sample Questions for Hour Exam 2

Fall 2018

This document is a collection of questions relevant to our upcoming exam that I have used on past Calculus 3 exams. I’ve included the original point value of each question, as an indication of how “big” I think each is (our exam will have a total of 50 points). All of the questions address material that might appear on our exam, but there are more questions here than will appear on it. I’ve included my solutions to each question, but I strongly recommend that you try to answer each question for yourself before looking at the solutions.

**Question 1** (10 Points). Let *z* be defined by the equation

Does *z* change more in response to a small change in the value of *x*, or to a small change in the value of *y*,around *x* = 1 and *y* = 3? Justify your choice in a sentence or two.

**Solution:** The amounts by which *z* changes in response to changes in *x* and *y* are given by *z*’s derivatives with respect to *x* and *y*, so find those derivatives and evaluate at point (*x*,*y*) = (1,3):

Thus at (1,3), and , so *z* changes faster with changes in *x* than with changes in *y*.

**Question 2** (10 Points). A function *f*(*x*,*y*) has level curves that are all straight lines of the form *y* = *x* + *c*, with different *c* values corresponding to different level curves. Suppose *f*(0,2) = 1. Find one other point (*x*,*y*) at which *f*(*x*,*y*) = 1. Explain your reasoning in a sentence or two.

**Solution.** Since *f*(0,2) = 1, we know that point (0,2) lies on the level curve for *f*(*x*,*y*) = 1. Furthermore, all points on this level curve must satisfy the equation *y* = *x* + *c*. Plugging in the (*x*,*y*) = (0,2) lets us find *c*, namely 2 = 0 + *c*, so *c* = 2. Any other point that also satisfies *y* = *x* + 2 lies on this level curve, and so represents another point at which *f*(*x*,*y*) = 1. Examples include (1,3), (2,4), etc.

**Question 3** (15 Points). Consider

Either find this limit, or show that it doesn’t exist.

**Solution:** Not seeing obvious ways to simplify the expression or otherwise find the limit, my first thought is to see if a 2-path test can show that it doesn’t exist. Approaching (1,1) along the path *x* = 1 produces

Approaching along the line *y* = 1 produces

Since the two limits differ,

must not exist.

**Question 4.** An ant is crawling on the surface of a sculpture, following the path

The ant starts crawling at point (0,1,0), where *t* = 0.

Part A (10 Points). Find a vector pointing in the direction the ant is moving at the instant when *t* = 2.

**Solution:** The unit tangent vector (or any other tangent vector) evaluated at *t* = 2 points in the direction the ant is moving.

So

Part B (15 Points). How far has the ant crawled between its starting point and its position at *t* = 2? (Use inches if you want units for your answer.)

**Solution:** The distance the ant crawls is given by the arc length of the curve from *t* = 0 to *t* = 2:

(The expression for the magnitude of ***r***′(*t*) comes from the solution to Part A.) Integrate this by using the substitution *u* = *t*2+1, *du* = 2*t* *dt*, i.e.,

**Question 5.** What is the domain of the function

Explain your answer in a sentence or two.

**Solution:** The function is defined for all values of *x* and *y* except *x* = 0 or *y* = 0, in which cases the denominator becomes 0. Thus the domain is all of ℝ2 except for the lines *x* = 0 and *y* = 0.

**Question 6.** Consider a function *g*(*x*,*y*), whose partial derivatives are

Part A (10 Points). Find all the second-order derivatives of *g*.

**Solution:** The second-order derivatives are

Part B (10 Points). Think about how *g* changes near the point (0,π). In particular, imagine comparing *g*(0,π) to *g*(0,π+ Δ), where Δ is some small quantity. About how much do you expect *g* to change by, in terms of Δ? (Or, in other words, roughly what do you expect the difference *g*(0,π+ Δ) - *g*(0,π) to be?)

**Solution:** Since Δ represents a small change in *y*, the change in *g* will be approximately with the derivative evaluated at (0,π). Doing this yields

**Question 7** (15 Points). Consider a function *f*(*x*,*y*) defined as

Show that this function is not continuous at (0,0).

(Note: I’m more interested in whether you can apply the definition of continuity than whether you remember it, so here it is: *f*(*x*,*y*) is continuous at point (*x*0,*y*0) if (1) f(*x*0,*y*0) is defined, (2) lim(*x*,*y*)⟶( *x*0,*y*0) *f*(*x*,*y*) is defined, and (3) lim(*x*,*y*)⟶( *x*0,*y*0) *f*(*x*,*y*) = f(*x*0,*y*0).)

**Solution:** Consider lim(*x*,*y*)→(0,0)*f*(*x*,*y*) along the paths *x* = 0 and *x* = *y*. Along the first of these paths

Along the second path

Since these limits are not equal, lim(*x*,*y*)→(0,0)*f*(*x*,*y*) does not exist, and so *f*(*x*,*y*) cannot be continuous at (0,0).

**Question 8** (10 Points). This table shows the values of function *f*(*x*,*y*) near (*x*,*y*) = (1,1). Based on this table, which of ∂*f*/∂*x* or ∂*f*/∂*y* do you think is larger at (1,1)? Explain your answer in a couple of sentences.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | | y | | | | |
| **0.98** | **0.99** | **1.0** | **1.01** | **1.02** |
| x | **0.98** | 10.58 | 10.59 | 10.60 | 10.61 | 10.62 |
| **0.99** | 10.78 | 10.79 | 10.80 | 10.81 | 10.82 |
| **1.0** | 10.98 | 10.99 | 11.00 | 11.01 | 11.02 |
| **1.01** | 11.18 | 11.19 | 11.20 | 11.21 | 11.22 |
| **1.02** | 11.38 | 11.39 | 11.40 | 11.41 | 11.42 |

**Solution:** Looking across the *x* = 1 row, *f*(1,*y*) seems to change by 0.01 unit for each 0.01-unit change in *y*. I thus estimate ∂*f*/∂*y* to be roughly 0.01/0.01 = 1 near (1,1). A similar analysis of the *y* = 1 column suggests that *f*(*x*,1) changes by 0.2 units per 0.01-unit change in *x*, so ∂*f*/∂*x* is roughly 0.2/0.01 = 20. I thus think ∂*f*/∂*x* is larger than ∂*f*/∂*y*.

**Question 9** (10 Points). Does the equation

describe a level curve for some function? If so, give an example of a function for which it’s a level curve, and say what value the function equals along that level curve. If not, explain briefly why not.

**Solution:** Yes, this equation describes a level curve for the function *f*(*x*,*y*) = *x*2 + *y*2 (among others), and the function equals 4 along that level curve.

**Question 10** (15 Points). Suppose you are painting a line that spirals around the surface of a cylindrical column in such a manner that *t* seconds after you start painting, the line has reached point

relative to some coordinate system that measures distances in feet. What are the coordinates of the end of the line when its total length is 10π feet?

**Solution:** I want to relate distance along the line to the coordinates of points on the line, which is what an arc length parameterization does. So I’ll start by finding one. Note that I’m integrating from a lower bound of *t* = 0, since I that’s when I start painting:

Solving for *t* in terms of *s*, *s* = 10*t* implies that *t* = *s*/10. Substituting this expression involving *s* (distance) in for *t* in the original parametric curve gives me my arc length parameterization:

Finally, plugging *s* = 10π into this equation gives me the desired coordinates: