Math 221 03
Prof. Doug Baldwin

Problem Set 7 — Derivative Applications 2

Complete by **Wednesday, April 17**
Grade by **Monday, April 22**

Purpose

This problem set develops your understanding of phenomena related to derivatives and their applications, including optimization, the Mean Value Theorem, and the end behavior of functions. By the time you finish this problem set you should be able to ...

- Recognize and interpret consequences of the Mean Value Theorem
- Find functions' limits at positive or negative infinity
- Solve optimization problems
- Use Mathematica or similar technology to explore problems related to differentiation
- Use previously studied differentiation and limit techniques in solving other problems.

Background

Chapter 4 of our textbook discusses applications of derivatives; this problem set focuses on material from sections 4.4, 4.6, and 4.7. We discussed the section 4.4 (the Mean Value Theorem) in class on March 29, and the remaining material between April 5 and 12.

Activity

Solve the following problems:

Question 1. (Based on OpenStax *Calculus, Volume 1*, section 4.4, exercise 190)

Part A. At 10:17 a.m., you are traveling 55 mph when you pass a police car that is stopped on the freeway. You pass a second stopped police car at 10:53 a.m., when you are also traveling 55 mph. The second police car is located 39 miles from the first one. If the speed limit is 60 mph, can the police cite you for speeding? Why or why not?

Part B. After being cited for speeding in Part A, you appeal the ticket on the grounds that your car is the new model with the high tech teleporter drive, which allows the car to move instantaneously from one place to another. Assuming the car really does have such a device, should the court dismiss your ticket? Why or why not?

Question 2. (OpenStax *Calculus, Volume 1*, section 4.6, exercise 294)

Find

$$\lim_{x \to -\infty} \frac{3x^3 - 2x}{x^2 + 2x + 8}$$

Question 3. I'm curious what range of values the function

$$f(x) = \frac{2x + 1}{3x}$$

takes on over the interval $(0, \infty)$. Answer my curiosity in the following steps...

Part A. Show that $f(x)$ is always decreasing over $(0, \infty)$.

Part B. Use limits and the fact that $f(x)$ is continuous on $(0, \infty)$ (you don't have to prove continuity, although see Part D below) to find the largest and smallest values that $f(x)$ takes on, or to show that there is no largest (and/or no smallest) value of $f(x)$. Use those values to say what range of values $f(x)$ takes on over the interval $(0, \infty)$. Hint: what does the fact that $f(x)$ is continuous and always decreasing tell you about where the largest and smallest values must occur?

Part C. Use Mathematica or similar technology to graph $f(x)$ over a large enough part of the interval $(0, \infty)$ to visually check that your answer to Part B is plausible.

Part D. For up to 2 points of extra credit, prove that $f(x)$ really is continuous on the interval $(0, \infty)$.

Question 4. (Based on OpenStax *Calculus, Volume 1*, Problem 341 in Section 4.7.)

Find the largest-volume right circular cylinder that fits inside a sphere of radius 1. Start by drawing a diagram of the situation.

Question 5. (Based on OpenStax *Calculus, Volume 1*, Problem 322 in Section 4.7.)

Two poles are connected by a wire that is also connected to the ground. The first pole is 20 ft tall and the second pole is 10 ft tall. There is a distance of 30 ft between the two poles. Use Mathematica or similar technology to determine where the wire should be anchored to the ground in order to minimize the amount of wire needed. See the textbook for a diagram of this situation.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.