Math 221 03
Prof. Doug Baldwin

# Problem Set 5 — Related Rates and Implicit Differentiation

Complete by **Wednesday, March 27**
Grade by **Monday, April 1**

## Purpose

This problem set mainly develops your understanding of implicit differentiation and how to solve related rates problems. It also reinforces your understanding of antiderivatives, particularly their use in solving initial value problems. By the time you finish this problem set you should be able to ...

- Find derivatives via implicit differentiation
- Solve related rates problems
- Solve initial value problems
- Use Mathematica or similar technology to graph implicit curves
- Use standard differentiation rules in finding derivatives

## Background

Section 3.7 of our textbook covers implicit differentiation, and we talked about it in classes between March 4 and 8. Section 4.1 discusses related rates problems, and we discussed them in class between March 8 and 14. Initial value problems were introduced via the online discussion that we did between February 27 and March 2.

## Activity

Solve the following problems:

**Question 1.** A car speeds past a speed trap. By the time the traffic officer gets a reading on the car's speed (time $t = 0$ seconds), it has already traveled 0.04 miles past the speed trap. Furthermore, it is accelerating in such a manner that $t$ seconds after this instant, it has a speed of $s = 85 + \frac{t}{20}$ miles per hour. How far away is the car by the time the traffic officer realizes what has happened, if realizing what happened takes 3 seconds?

You may use a calculator for the numeric calculations in this exercise. To convert miles per hour into miles per second, divide miles per hour by 3600.

**Question 2.** (A variation on OpenStax *Calculus, Volume 1*, section 3.7, exercise 262)

Use implicit differentiation to find $\frac{dy}{dx}$ given that $x^2y = y - 7$. Also use Mathematica or similar technology to plot this curve in the region $-10 \leq x \leq 10$, $-10 \leq y \leq 10$.

**Question 3.** (OpenStax *Calculus, Volume 1*, section 4.1, exercise 16)

The side of a cube increases at a rate of $\frac{1}{2}$ m/sec. Find the rate at which the volume of the cube increases when the side of the cube is 4 m.

**Question 4.** (Inspired by OpenStax *Calculus, Volume 1*, section 4.1, exercises 10 and 11. Both strike me as a little unusual as related rates problems, but they are good exercises in calculus- and geometry-based problem-solving.)

A 6-ft tall person walks away from a 10-ft lamppost at a constant rate of 3 ft/sec. See the picture in our textbook for a visual presentation.

**Part A.** What is the rate at which the tip of the person's shadow moves away from the *person* when the person is 10 ft away from the pole?

**Part B.** What is the rate at which the tip of the person's shadow moves away from the *pole* when the person is 10 ft away from the pole?

## Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.