

Problem Set 3 — Introduction to Derivatives

Complete by **Sunday, February 24**
Grade by **Wednesday, February 27**

Purpose

This problem set reinforces basic ideas of derivatives. Since derivatives are really limits, it also gives you some more practice finding limits. In particular, by the time you finish this problem set you should be able to ...

- Find derivatives using differentiation rules
- Find derivatives using the limit definition of the derivative
- Find derivatives using Mathematica or equivalent technology
- Relate the appearance of functions' graphs to their derivatives
- Evaluate limits using limit laws and algebra.

Background

This problem set is based on material in sections 3.1 through 3.3 of our textbook. We discussed this material in class between February 14 and 18.

Activity

Solve the following problems:

Question 1. Use the limit definition of the derivative to find the derivatives of the following functions. Then confirm your answers by finding the same derivatives with Mathematica:

1. $f(x) = 3x - 4$

2. $f(x) = \frac{x}{x+1}$

3. $g(t) = \sqrt{t^2 - t}$

Question 2. Use differentiation rules to find the derivatives of the following functions. Then confirm your answers by finding the same derivatives with Mathematica.

1. $f(t) = \frac{t^3}{3} + 2t^2 - t + 5$

2. $g(x) = \frac{1}{x^2}$

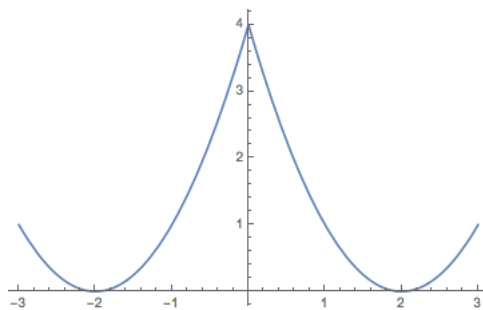
3. $s(t) = \frac{t^2-1}{6t}$

4. $f(z) = (z^3 + 3z^2)(z - 1)$

Question 3. Using the plot of function $f(x)$ below, identify. . .

1. A value of x at which $f'(x)$ is negative
2. A value of x at which $f'(x)$ is zero
3. A value of x at which $f'(x)$ is positive
4. A value of x at which $f'(x)$ is undefined.

Be prepared to explain why you believe the derivative has the property in question at the values you pick.



Question 4. Use the limit definition of the derivative to prove the constant rule for differentiation (i.e., prove that if $f(x) = c$ for some constant c , then $f'(x) = 0$, our textbook's Theorem 3.2).

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along. Be sure to also bring either a print-out of your Mathematica work, or a computer with your work loaded into Mathematica.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.