Math 221 03
Prof. Doug Baldwin

Problem Set 1 — Limits

Complete by **Sunday, February 3**
Grade by **Wednesday, February 6**

Purpose

This problem set develops your conceptual understanding of limits, and your ability to find limits via limit laws and algebra. The problem set also provides a chance to practice/review some algebra and trigonometry. By the time you finish this problem set you should be able to ...

- Use limit laws and algebra to find limits of functions
- Understand relations that can exist between functions' values and their limits, and identify situations in which these relations exist.

Background

This problem set is based on material in sections 2.2 and 2.3 of our textbook. We will discuss this material in class between January 28 and 31.

Although this problem set is an opportunity to dust off some of your algebra and trigonometry skills, I have not planned specific times in class to review those things. I will, however, be more than happy to answer questions about them, either in class or out of class. The Math Learning Center is also a good place to get help with such questions.

Activity

Solve the following problems:

Question 1. Use limit laws and algebra to find the following limits:

1. $$\lim_{x \to 2} \frac{x-2}{x^2-2x}$$

   (Exercise 94 in section 2.3 of the textbook).

2. $$\lim_{h \to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h}$$

   (Inspired by Exercise 98 in section 2.3 of the textbook).

3.
$$\lim_{x \to \frac{\pi}{4}} \frac{\cos(2x)}{\cos x - \sin x}$$
(Hint: review your trigonometric formulas — there's a list of them in appendix C.3 in the textbook. You can also assume — we'll see proofs in a couple of weeks — that for all real numbers a, $\lim_{x \to a} \sin x = \sin a$ and $\lim_{x \to a} \cos x = \cos a$).

4.
$$\lim_{x \to -3} \frac{\sqrt{x+4} - 1}{x+3}$$
(Exercise 102 from section 2.3 of the textbook).

Question 2. Consider the function
$$f(t) = \frac{(1+2t)^2 - 1}{t}$$

Part A. Give an example of a value a for which $f(a)$ is undefined, but $\lim_{t \to a} f(t)$ is defined. What is the value of the limit?

Part B. Give an example of a value b for which $f(b)$ is defined and $\lim_{t \to b} f(t) = f(b)$.

Question 3. Sketch a graph over the interval $[0, 4]$ of a function $f(x)$ that has the following features:

1. $f(0) = f(4) = 0$.
2. $f(1)$ is defined and equals $\lim_{x \to 1} f(x)$.
3. $f(2)$ is undefined, but $\lim_{x \to 2} f(x)$ is defined.
4. $f(3)$ is defined, and $\lim_{x \to 3} f(x)$ is also defined, but $f(3) \neq \lim_{x \to 3} f(x)$.

Note that I'm only asking you to sketch a graph of such a function, not find an equation that defines it (although giving an equation as well as the graph would be an example of something that goes beyond what I expect on this problem set).

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.