Math 221 03 — Sample Final Exam Questions

May, 2019

General Directions. These are some questions from a past calculus 1 test. I've indicated the original point values of each question to give you a sense of how difficult and/or important I considered each one. Most are similar to questions I could ask this semester, although there's one that deals with volume, which we won't cover in enough depth to make it fair to ask about on our test. I've included that question anyhow, because it is still a chance to practice related concepts and methods.

Question 1. (10 points) Here are the graphs of three functions over some interval. Which function has the largest definite integral over this interval? Explain your choice in a sentence or two.

Question 2. My car tells me about its fuel efficiency in two ways: a display that shows the average miles per gallon the car has gotten since the last time I filled the gas tank, and another that shows the instantaneous miles per gallon the car is getting now.

Part A. (10 points) Can the miles per gallon measure of fuel efficiency be thought of as a derivative? If so, what is it a derivative of, with respect to what? If not, why not?

Part B. (10 points) The other day I noticed that the average miles per gallon read-out was equal to a value I had seen on the instantaneous one the previous day. Should it surprise me that this sometimes happens? Explain why or why not in a sentence or two.

Question 3. (15 points) (There probably won't be volume questions on our final, though I do expect to talk about them in class.) Imagine rotating the graph of

$$y = \frac{x^3}{27}$$

between $x = -3$ and $x = 3$ around the y axis, to form an hourglass shaped volume (contained between the y axis and the rotated graph). Find the volume of this "hourglass."

Question 4. (15 points) Phineas Phoole is trying to find

$$\int (2\tan(2x)\sec(2x))^2\,dx$$

Martha the Mathematician points out that there is a u-substitution that turns this integral into

$$\int 2u^2\,du$$

Identify the u and du that make this possible.

Question 5. (15 points) Geneseo Widget Works, Inc. makes widgets. If they employ n people to work together on one widget, the time to make the widget is

$$t(n) = \frac{2}{n} + \frac{n^2}{8}$$

How many people should Geneseo Widget Works have work together on a widget in order to minimize the time it takes to make it?

Question 6. (15 points) A circular oil slick has a uniform and constant thickness of 0.1 cm. Oil is seeping into the slick at a constant rate of 1 cm^3 per second. How fast is the circumference of the slick growing when the radius is 100 cm?

Question 7. (15 points) In the magical kingdom of Zrr, materials that we normally think of as inanimate have free will and emotions. Among other things, this means that Zrr engineers are concerned for how enthusiastic their building materials are about the structures they become part of — if you're building a Zrr road for example, you would prefer that it's built out of materials that are enthusiastic about carrying traffic rather than ones that would rather be doing something else. Sometimes different parts of an object have different levels of enthusiasm, in which case the overall enthusiasm of the object is the sum of each part's enthusiasm times its volume. Enthusiasm is measured in units of "wows," so, for example, if a parking area consists of a 2 cubic meters of asphalt with 1 wow of enthusiasm, and 4 cubic meters with 3 wows, the overall enthusiasm of the parking area is $2 \times 1 + 4 \times 3 = 14$ cubic-meter-wows (note that overall enthusiasm has different units from simple enthusiasm).

Suppose a Zrr math student has a thin cylindrical rod of radius r laid out along the x axis between points $x = a$ and $x = b$. At any position x between these bounds, the rod's enthusiasm is $f(x)$, for some function f. Use a Riemann sum to show that the overall enthusiasm of the rod can be calculated as a definite integral. Show what the integral looks like, although you don't have enough information in this question to actually evaluate it.

Question 8. (15 points) Identify one value of a at which

$$\lim_{x \to a} \frac{1}{(\sin x)^3}$$

does not exist. Explain why the limit does not exist at your a in a sentence or two.