

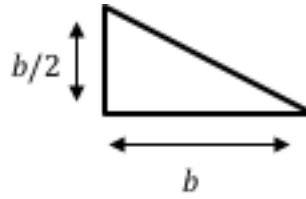
Math 221 03 — Sample Exam 2

April 7, 2019

General Directions. These are some questions I have asked on the second exam other times I have taught this course. They won't align perfectly with what I'll ask about this time (e.g., I probably won't ask about the Mean Value Theorem), but they are hopefully useful practice nonetheless. I've listed the original point values of the questions, which may give you some sense of the relative importance I attached to each, but there are more questions here, and thus more points, than I would actually put on a 50 minute test. I've also marked each question with what I consider to be its main topic, and sometimes other brief notes.

Question 1. (10 points) (Mean Value Theorem.) Geneseo's College Magic Carpet flew from Geneseo to Rochester, a distance of 30 miles, in 15 minutes. Knowing only that when the carpet moves from place to place it does so in a continuous motion, and that its speed is a differentiable function of time, identify a speed that you know the carpet was traveling at at some instant during its trip. You do not have to know when the carpet was traveling at this speed, but do say in a sentence or two how you know the carpet was traveling at it sometime. Do not assume the carpet started or ended with speed 0 — it may have had a running start or coasted to a stop after arrival.

Question 2. (15 points) (Related Rates.) Imagine a right triangle whose height is always half its base, like this:



The triangle is expanding in such a way that the length of the base increases by 3 cm every second (and so the height also increases to stay half the length of the base). How fast is the triangle's area increasing when the base is 6 cm long?

Question 3. (10 points) (Chain Rule, but unlike chain rule problems we did this semester.) Suppose that y is computed as $y = \frac{3}{2}f(x)^2 + f(x)$, for some unknown function f . Further suppose that $f(2) = 1$ and $\frac{dy}{dx} = 12$ when $x = 2$. Find $f'(2)$.

Question 4. (15 points) (Extreme Values.) Consider the function

$$f(x) = 2x + \frac{1}{x^2}$$

Show that f has no absolute maximum over the interval $(0, \infty)$.

Question 5. (10 points) (Implicit Differentiation.) Given the following relationship, find $\frac{dy}{dx}$:

$$\frac{x + 3y^2}{x^2 + y^2 + 1} = 1$$

Question 6. (15 points) (Related Rates.) An anchored boat has drifted downstream from its anchor, thus:



The crew decides to correct this drift by rowing back upstream, reeling in the anchor chain as they go. If the water is a uniform 30 feet deep, the rowers move the boat at a steady rate of 10 feet per minute, and the anchor reeler keeps the chain taut, how fast are they reeling in anchor chain when the boat is 40 feet downstream from the anchor?