Math 221 — Hour Exam 1

October 5, 2017

General Directions. This is an open-book, open-notes, open-computer test. However, you may not communicate with any person, except me, during the test. You have the full class period (50 minutes) in which to do the test. Put your answer to each question in the space provided (use the backs of pages if you need more space). Be sure to **show your work**! I give partial credit for incorrect answers if you show correct steps leading up to them; conversely, I do not give full credit even for correct answers if it is not clear that you understand where those answers come from. Good luck.

This test contains 5 questions on 5 pages.

Question 1 (5 Points). Give an example of a function $f(x)$ whose derivative is $9x^2 - 2x$.

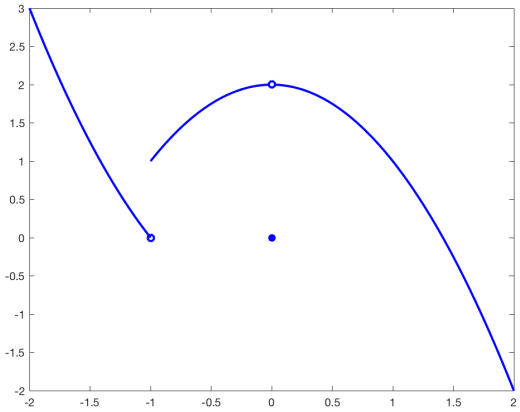
Question 2 (15 Points). If $f(x)$ is a function, its "symmetric difference quotient" is a fraction of the form

$$\frac{f(x+h)-f(x-h)}{2h}$$

where h is a number, typically small. Show that for $f(x) = x^2 - 3$, the limit of f's symmetric difference quotient as h approaches 0 equals f's derivative*.

* People really do use symmetric difference quotients to estimate derivatives, although you have to be careful doing so because there are some functions for which the limit of the symmetric difference quotient is not equal to the derivative.

Question 3 (10 Points). Here is a graph of a function $g(x)$. Identify all the numbers a in the open interval (-2, 2) where $\lim_{x \to a} g(x)$ does not exist. For each such a, explain in about one sentence why the limit doesn't exist.



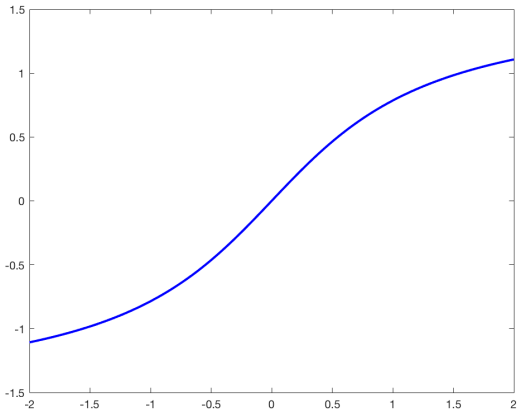
Question 4 (15 Points). The calculations that produce visually correct perspective in computer graphics (e.g., in the images in a first-person video game or computer-generated movie) cause the apparent height of an object distance r from the viewer to be proportional to $1/r$. Specifically, if h represents the apparent height of an object whose actual height is H, the relationship is

$$h = \frac{kH}{r}$$

where k is a constant that depends on the particular graphics system.

In a hypothetical video game, a character 100 feet away from you is walking toward you at a speed of 1 foot per second. Further suppose the character's height is constant and the game is designed so that kH = 10 square feet ("square feet" sounds bizarre, but the way k is derived it has units of distance, and H is a distance). How fast is the character's apparent height (h in the above equation) changing?

Question 5 (5 Points). Here is a graph of a certain function near $x = 0$:



At roughly which of the x values in the graph is this function's derivative largest? Explain your choice in about one sentence.