Math 221 06
Prof. Doug Baldwin

Problem Set 8 — Applications of Derivatives

Complete by **Tuesday, October 29**
Grade by **Friday, November 1**

Purpose

This problem set mainly reinforces your understanding of several applications of derivatives, namely ones related to local and absolute extreme values, and linear approximations. The problem set also develops your understanding of the Mean Value Theorem. By the time you finish this problem set you should be able to . . .

- Find linear approximations and use them to estimate the value of nonlinear functions
- Use derivatives to find local extreme values
- Use derivatives to find absolute extreme values
- Use the Mean Value Theorem to make inferences about derivatives
- Use Mathematica as a tool for understanding and solving calculus problems.

Background

Our textbook discusses linear approximation in section 4.2, and we discussed it in class on October 17. Extreme values are in section 4.3, and were the subject of our October 18 and 21 classes. The Mean Value Theorem is section 4.4, and was covered in class on October 23.

Activity

Solve the following problems:

Question 1. (Exercise 52 in section 4.2 of OpenStax *Calculus, Volume 1 for SUNY Geneseo*)

Find a linear approximation to $f(x) = \tan x$ near $x = \frac{\pi}{4}$.

Question 2. (Based on exercise 59 in section 4.2 of OpenStax *Calculus, Volume 1 for SUNY Geneseo*)

Use linear approximation to estimate the value of $15.99^{\frac{1}{4}}$ (15.99 raised to the $\frac{1}{4}$ power). Decide for yourself what function to do an approximation of, and near what value. You may use a calculator for the resulting numeric calculations. You should also use a calculator to see how close your linear estimate is to the machine-calculated value.

Question 3. (Exercise 90 in section 4.3 of OpenStax *Calculus, Volume 1 for SUNY Geneseo*)

Every 2nd degree polynomial, i.e., every function of the form $y = ax^2 + bx + c$, has exactly one maximum or minimum. You may also have heard that this maximum or minimum occurs at $x = \frac{-b}{2a}$. Use calculus to derive this result.

Question 4. In class on October 21 we saw a particularly steep ski trail at the Mt. Doom ski resort. Mt. Doom has an even worse trail, whose elevation (h) as a function of ground-level distance from the bottom end of the trail (x) is given by the equation

$$h(x) = x^4 - 4x^3 + 2x^2 + 4x, \quad 0 \leq x \leq \pi$$

Use Mathematica to help you answer the following questions. In answering these questions, it may help to keep in mind that skiers going down this trail move from right to left on a graph, i.e, from larger values of x towards smaller ones.

Part A. Plot the height profile (i.e., height versus x) of the ski slope.

Part B. What are the highest and lowest elevations on this ski slope, and at what x values do they occur? Give answers as decimal numbers with at least 4 decimal digits precision.

Part C. What is the steepest downhill slope on the ski trail, and at what x value does it occur? Give your answer as a decimal number with at least 4 decimal digits precision. Also give your answer as an angle in degrees.

Question 5. (Based on exercise 190 in section 4.4 of OpenStax *Calculus, Volume 1 for SUNY Geneseo*)

Part A. At 10:17 a.m., you are traveling 55 mph when you pass a police car that is stopped on the freeway. You pass a second stopped police car at 10:53 a.m., when you are also traveling 55 mph. The second police car is located 39 miles from the first one. If the speed limit is 60 mph, can the police cite you for speeding? Why or why not?

Part B. After being cited for speeding in Part A, you appeal the ticket on the grounds that your car is the new model with the high tech teleporter drive, which allows the car to move instantaneously from one place to another. Assuming the car really does have such a device, should the court dismiss your ticket? Why or why not?

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the “Grade By” date above.