Math 221 06
Prof. Doug Baldwin

Problem Set 7 — Applications of the Chain Rule

Complete by **Wednesday, October 23**
Grade by **Monday, October 28**

Purpose

This problem set reinforces your understanding of two key applications of the chain rule: implicit differentiation and related rates. By the time you finish this problem set you should be able to ...

- Use implicit differentiation to find derivatives of variables defined by implicit equations
- Solve related rates problems
- Plot implicit equations with Mathematica.

Background

Our textbook discusses implicit differentiation in section 3.8, and we discussed it in class on October 3. Section 4.1 of the textbook discusses related rates problems, and we talked about them in class on October 11 and 16. I'll talk about plotting implicit functions with Mathematica in class on October 17.

Activity

Solve the following problems:

Question 1. (A variation on exercise 302 in section 3.8 of OpenStax *Calculus, Volume 1 for SUNY Geneseo*)

Use implicit differentiation to find $\frac{dy}{dx}$ given that $x^2y = y - 7$. Also use Mathematica to plot this curve in the region $-10 \leq x \leq 10$, $-10 \leq y \leq 10$.

Question 2. (Based on exercise 320 in section 3.8 of OpenStax *Calculus, Volume 1 for SUNY Geneseo.*)

Find the equation of the line tangent to the graph of the equation

$$\sin^{-1}x + \sin^{-1}y = \frac{\pi}{6}$$

at point $(0, \frac{1}{2})$. If you aren't sure how to find the equation for a tangent line, ask in class – lots of other people in the room will probably be silently grateful that you did!

Also use Mathematica to graph this equation in the region $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$. See if you can explain why the curve doesn't extend all the way to $x = -1$ or $y = -1$.

Question 3. (Exercise 16 in section 4.1 of OpenStax *Calculus, Volume 1 for SUNY Geneseo*)

The side of a cube increases at a rate of $\frac{1}{2}$ m/sec. Find the rate at which the volume of the cube increases when the side of the cube is 4 m.

Question 4. (Inspired by exercises 10 and 11 in section 4.1 of OpenStax *Calculus, Volume 1 for SUNY Geneseo*. The pair is a little unusual as related rates problems, but they are good exercises in calculus- and geometry-based problem-solving.)

A 6-ft tall person walks away from a 10-ft lamppost at a constant rate of 3 ft/sec. See the picture in our textbook for a visual presentation.

Part A. What is the rate at which the tip of the person's shadow moves away from the *person* when the person is 10 ft away from the pole?

Part B. What is the rate at which the tip of the person's shadow moves away from the *pole* when the person is 10 ft away from the pole?

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.