Math 221 06
Prof. Doug Baldwin

Problem Set 6 — The Chain Rule

Complete by **Tuesday, October 15**
Grade by **Friday, October 18**

Purpose

This problem set reinforces your understanding of the chain rule. In the process, it also develops your ability to work with derivatives of various transcendental functions. In particular, by the time you finish this problem set you should be able to ...

- Use the chain rule to find derivatives
- Find derivatives of exponential, logarithmic, and inverse trigonometric functions
- Find antiderivatives involving exponential, logarithmic, and inverse trigonometric functions
- Use exponential, logarithmic, and inverse trigonometric functions in Mathematica.

Background

This problem set mostly draws on section 3.6 of our textbook. However, derivatives of the transcendental functions used in this problem set are presented in sections 3.7 and 3.9. We discussed all of this material in class between September 30 and October 9.

Activity

Solve the following problems:

Question 1. Given the following definitions of y, find $\frac{dy}{dx}$ by hand. Then use Mathematica to find $\frac{dy}{dx}$ and confirm that both methods get equal results. These problems come from or at least are inspired by OpenStax *Calculus, Volume 1 for SUNY Geneseo*:

1. $y = x^2 \cos^4 x$ (exercise 234 in section 3.6).
2. $y = (6 + \sec(\pi x^2))^2$ (inspired by exercise 236 in section 3.6).
3. $y = \sqrt{e^{2x} + 2x}$ (exercise 334 in section 3.9)

Question 2. (Exercise 238 in section 3.6 of *Calculus, Volume 1 for SUNY Geneseo*)
Let $y = (f(x))^3$ and suppose that $f'(1) = 4$, and that $\frac{dy}{dx} = 10$ when $x = 1$. What is $f(1)$?

Question 3. Find the derivative of each of the following functions by hand. Then check your answers by finding the derivatives with Mathematica. The derivatives will be easier to evaluate by hand if you use algebraic rules related to exponential functions or logarithms to simplify the functions before taking derivatives (but for the Mathematica part, you can just give the functions to Mathematica unsimplified and let it do its thing with them).

- $g(t) = \frac{xe^{x+1}}{e^{x-1}}$
- $f(a) = \ln\left((x\sin^{-1}x)^{\frac{3}{5}}\right)$
- $r(t) = (t+1)^{2t-1}$

Question 4. Find the following antiderivatives:

- $\int \frac{2\,dx}{3x}$, assuming $x > 0$
- $\int \frac{dt}{1+t^2}$
- $\int 4e^x - 3e^{-x}\,dx$

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.