Math 221 06
Prof. Doug Baldwin

Problem Set 4 — Introduction to Derivatives

Complete by **Thursday, September 26**
Grade by **Tuesday, October 1**

Purpose

This problem set develops basic ideas of derivatives and rules for finding them. In particular, by the time you finish this problem set you should be able to ...

- Find derivatives using the limit definition of the derivative
- Find limits using algebra and limit laws
- Find derivatives using differentiation rules
- Find derivatives using Mathematica
- Relate the appearance of functions' graphs to their derivatives
- Find antiderivatives using differentiation rules "in reverse"

Background

This problem set mostly uses ideas from sections 3.1 through 3.3 of our textbook. Our textbook discusses antiderivatives later, but I plan to introduce the idea in class on September 19. I also expect to demonstrate derivatives in Mathematica on the 19th. More generally, we discussed (or will discuss) the material for this problem set in classes between September 16 and 20.

Activity

Solve the following problems:

Question 1. Use the limit definition of the derivative to find the derivatives of the following functions. Then confirm your answers by finding the same derivatives with Mathematica:

1. $f(x) = 3x - 4$
2. $f(x) = \frac{x}{x+1}$
3. $g(t) = \sqrt{t^2 - t}$

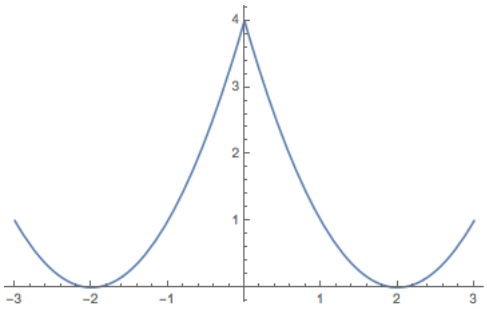
Question 2. Use differentiation rules to find the derivatives of the following functions. Then confirm your answers by finding the same derivatives with Mathematica.

1. $f(t) = \frac{t^3}{3} + 2t^2 - t + 5$
2. $s(t) = \frac{t^2-1}{6t}$
3. $f(z) = (z^3 + 3z^2)(z - 1)$

Question 3. Using the plot of function $f(x)$ below, identify...

1. A value of x at which $f'(x)$ is negative
2. A value of x at which $f'(x)$ is zero
3. A value of x at which $f'(x)$ is positive
4. A value of x at which $f'(x)$ is undefined.

Be prepared to explain why you believe the derivative has the property in question at the values you pick.



Question 4. Find the general antiderivative of each of the following functions:

1. $f(x) = 4x$
2. $h(t) = 6t^2 - \frac{t}{2} + 9$
3. $f(y) = 3y^5 + 5y^4 + 3y$

Question 5. Use the limit definition of the derivative to prove the difference rule, i.e., to prove that if $f(x) = g(x) - k(x)$ where $g(x)$ and $k(x)$ are both differentiable, then $f'(x) = g'(x) - k'(x)$.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.