Math 221 06
Prof. Doug Baldwin

Problem Set 3 — Limit Variations

Complete by **Tuesday, September 17**
Grade by **Friday, September 20**

Purpose

This problem set develops your understanding of variations on the basic idea of a limit, for example one-sided limits or infinite limits. In particular, by the time you finish this problem set you should be able to ...

- Find one-sided limits, using algebraic simplifications and/or limit laws as appropriate
- Use one-sided limits to determine whether a two-sided limit exists, and if so what it is
- Find infinite limits, using algebraic simplifications, limit laws, and the book's infinite limit theorem as appropriate
- Recognize when a function is and is not continuous

Background

This problem set uses ideas from sections 2.2 through 2.4 of our textbook. We discussed this material in classes between September 11 and 13.

Activity

Solve the following problems:

Question 1. Find

$$\lim_{x \to 1^-} \frac{x^2 - 1}{|x - 1|}$$

and

$$\lim_{x \to 1^+} \frac{x^2 - 1}{|x - 1|}$$

Does $\lim_{x \to 1} \frac{x^2-1}{|x-1|}$ exist? If so, what is it? If not, why not?

Question 2. Consider the function $f(x)$ defined piecewise by

$$f(x) = \begin{cases} x^3 + 1 & \text{if } x < 0 \\ 1 - x^2 & \text{if } x \geq 0 \end{cases}$$

Part A. Find $\lim_{x \to 0^-} f(x)$.

Part B. Find $\lim_{x \to 0^+} f(x)$.

Part C. Does $\lim_{x \to 0} f(x)$ exist? If so, what is its value? If not, why not?

Part D. Is $f(x)$ continuous at $x = 0$. Explain why or why not.

Question 3. (Based on exercise 154 in section 2.4 of Openstax *Calculus Volume 1.*)

Using the graph in exercise 154 (at the top of page 192 in our PDF of the book), say whether the function $f(x)$ is or is not continuous at each of the following x values:

1. $x = -2$
2. $x = -1$
3. $x = 0$
4. $x = 1$
5. $x = 2$

For each value at which the function is discontinuous, say what part of the definition of continuity is violated, and identify the discontinuity as a jump, infinite, or removable discontinuity.

Question 4. Newton's law of gravitation implies that the acceleration due to gravity at the surface of a star of mass m (measured in kilograms) is

$$a = G\frac{m}{r^2}$$

where r is the radius of the star in meters and G is a constant approximately equal to 6.67×10^{-11} $\mathrm{Nm}^2/\mathrm{kg}^2$.

Imagine that a spaceship (evidently a very hardy one) is skimming the surface of a star that starts collapsing. The star collapses in such a way that its mass remains constant, but gets packed into an ever smaller radius, and throughout the collapse the spaceship keeps flying right along the surface of the star. Calculate the acceleration due to the star's gravity on the spaceship in the limit as the star's radius approaches 0.

To what extent do you think you can rely on this mathematical result as a model of physical reality? (For brownie points, but maybe not actual grade points, what is a star called if it collapses to the point that its radius approaches 0?)

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.