Math 221 06
Prof. Doug Baldwin

Problem Set 1 — Limits

Complete by **Sunday, September 8**
Grade by **Wednesday, September 11**

Purpose

This problem set familiarizes you with the concept of a limit, and with some of the mathematical rules and definitions related to limits. In particular, by the time you finish this problem set you should be able to ...

- Informally recognize limits in tabular presentations of functions
- Informally recognize limits in graphical presentations of functions
- Relate terms in the formal definition of limit to concrete properties of functions near limits
- Use limit laws to evaluate limits
- Plot functions with Mathematica.

Background

This problem set draws on sections 2.2, 2.3, and 2.5 of our textbook. I plan to discuss this material in classes between August 30 and September 5. I will introduce Mathematica and how to use it to plot functions in class on August 30.

Activity

Solve the following problems:

Question 1. (Expanding on Exercises 32 through 34 in section 2.2 of *Calculus Volume 1 for SUNY Geneseo*)

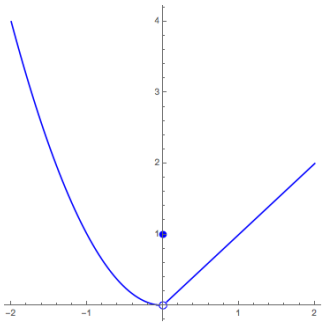
Part A. Generate a table of values of the function $f(x) = (x+1)^{\frac{1}{x}}$ for $x = -0.01$, -0.001, -0.0001, -0.00001, 0.00001, 0.0001, 0.001, and 0.01. You may use a calculator or other technology in answering this question (a spreadsheet can make very short work of the calculations).

Part B. Based on this table, estimate the value of $\lim_{x\to 0}(x+1)^{\frac{1}{x}}$.

Part C. Finally, use Mathematica to plot $(x+1)^{\frac{1}{x}}$ for x ranging from -0.1 to 0.1; explain why this graph is or is not consistent with the limit you estimated from the table.

Part D. (Optional) To go a bit past my expectations on this question, $\lim_{x\to 0}(x+1)^{\frac{1}{x}}$ turns out to be an often-used mathematical constant. What constant is it (give the name of the constant).

Question 2. Here is a graph of a function $f(x)$:



Part A. Based on this graph, what do you estimate $\lim_{x\to -1} f(x)$ to be?

Part B. Based on this graph, what do you estimate $\lim_{x\to 0} f(x)$ to be?

Question 3. Sketch a graph over the interval $[0, 4]$ of a function $f(x)$ that has the following features:

1. $f(0) = f(4) = 0$.
2. $f(1)$ is defined and equals $\lim_{x\to 1} f(x)$.
3. $f(2)$ is undefined, but $\lim_{x\to 2} f(x)$ is defined.
4. $f(3)$ is defined, and $\lim_{x\to 3} f(x)$ is also defined, but $f(3) \neq \lim_{x\to 3} f(x)$.

Note that I'm only asking you to sketch a graph of such a function, not find an equation that defines it (although giving an equation as well as the graph would be an example of something that goes beyond what I expect on this problem set).

Question 4. (An extension of exercise 84 in section 2.3 of *Calculus Volume 1 for SUNY Geneseo.*)

Part A. Use limit laws to find

$$\lim_{x\to 1} \frac{x^3 + 3x^2 + 5}{4 - 7x}$$

Show each step in applying the limit laws.

Part B. Use Mathematica to plot

$$\frac{x^3 + 3x^2 + 5}{4 - 7x}$$

over a small interval around $x = 1$ (I used $0.75 \leq x \leq 1.25$) to verify that the limit you found in Part A is correct.

Question 5. (An extension of exercise 184 in section 2.5 of *Calculus Volume 1 for SUNY Geneseo.*)

The textbook shows a graph of a function $f(x)$, and states that $\lim_{x\to 3} f(x) = 2$. See the textbook for the exact graph.

Part A. Estimate a value of δ that makes the precise definition of limit hold for $\epsilon = 1.5$.

Part B. Estimate a value of ϵ that makes the precise definition of limit hold for $\delta = 1$.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along. Be sure to also bring either a print-out of your Mathematica work, or a computer with your work loaded into Mathematica.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.