Math 221 06
Prof. Doug Baldwin

Last (!) Problem Set — Integration

Complete by **Wednesday, December 4**
Grade by **Monday, December 9**

Purpose

This problem set develops your ability to calculate integrals and use them to solve area and volume problems. In particular, by the time you finish this problem set you should be able to ...

- Use substitution to evaluate indefinite integrals
- Use substitution to evaluate definite integrals
- Describe and solve problems related to areas and volumes in terms of integration
- Use Mathematica to solve integration problems.

Background

This exercise is mainly based on material in sections 5.5, 6.1, and 6.2 of our textbook. We discussed, or will discuss, that material in class between November 20 and December 4.

Activity

Solve the following problems:

Question 1. Izzy Newton, great-to-the-umpteenth grandchild of Sir Isaac, has found some of Sir Isaac's long-lost notes in a chest in the attic. Although the notes are damaged in places by age, they are largely still readable. Translated into modern language and notation, one of them reads as follows, with damaged portions indicated by "???"

> To find the antiderivative
> $$\int 2x(e^{x^2} - 1)\,dx$$
> we can use the substitution
> ???
> Replacing u in this last equation with its definition in terms of x, we conclude that
> $$\int 2x(e^{x^2} - 1)\,dx = e^{x^2} - x^2 + C$$

Note that "this last equation" in Sir Isaac's last line refers to an equation lost in the damaged section. What substitution did Sir Isaac use, and what else do you think was in the damaged part of the note?

Question 2. Evaluate each of the following definite or indefinite integrals. After evaluating each by hand, check your answer by evaluating the integral with Mathematica.

Part A. (Exercise 292 in section 5.5 of OpenStax *Calculus Volume 1 for SUNY Geneseo*)

$$\int_0^1 x\sqrt{1-x^2}\,dx$$

Part B. (Exercise 296 in section 5.5 of OpenStax *Calculus Volume 1 for SUNY Geneseo*)

$$\int_0^{\frac{\pi}{4}} \sec^2\Theta \tan\Theta\, d\Theta$$

Part C. (Exercise 278 in section 5.5 of OpenStax *Calculus Volume 1 for SUNY Geneseo*)

$$\int \sin^2 x \cos^3 x\, dx$$

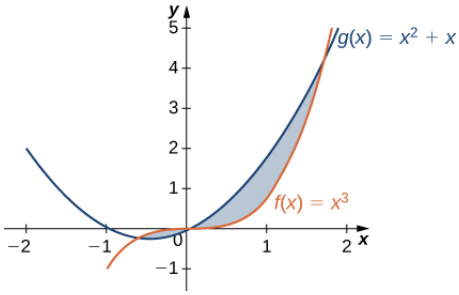
Hint: remember that $\cos^2 x = 1 - \sin^2 x\, dx$.

Question 3. Prove the following extension to the power rule for antiderivatives: For all constants $n \neq -1$ and k,

$$\int (x+k)^n\, dx = \frac{1}{n+1}(x+k)^{n+1} + C$$

Question 4. (Based on exercise 3 in section 6.1 of OpenStax *Calculus Volume 1 for SUNY Geneseo*)

Use Mathematica to find the area between the curves $y = x^3$ and $y = x^2 + x$, as shown in the following graph:



Question 5. (Exercise 58 in section 6.2 of OpenStax *Calculus Volume 1 for SUNY Geneseo*)

Derive the formula for the volume of a sphere ($V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere) by using the slice method.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.