Math 221 06
Prof. Doug Baldwin

Problem Set 12 — Integrals

Complete by **Thursday, November 21**
Grade by **Tuesday, November 26**

Purpose

This problem set develops your ability to calculate with and reason about integrals. By the time you finish this problem set you should be able to ...

- Use knowledge of area from geometry to reason about integrals.
- Use algebraic properties of integrals to reason about them.
- Use the Fundamental Theorem of Calculus to reason about integrals and their relationship to derivatives.
- Calculate integrals using Mathematica.

Background

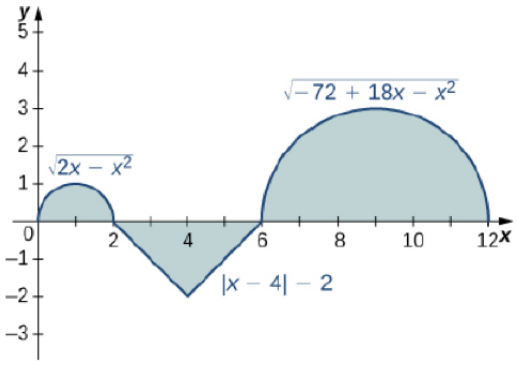
This exercise is mainly based on material in sections 5.2 and 5.3 of our textbook. We discussed that material in class between November 11 and 18. We looked at Mathematica's "Integrate" function for evaluating integrals in class on November 14.

Activity

Solve the following problems:

Question 1. (Based on Exercise 72 in section 5.2 of Openstax *Calculus, Volume 1 for SUNY Geneseo.*)

Consider the function, $f(x)$, defined by the following graph. Note that the graph also contains definitions of functions that produce each piece of the graph — in effect, giving you a piecewise definition of $f(x)$.



Use formulas for the areas of semicircles and triangles to answer the following....

Part A. Find the integral of $f(x)$ from $x = 0$ to $x = 12$.

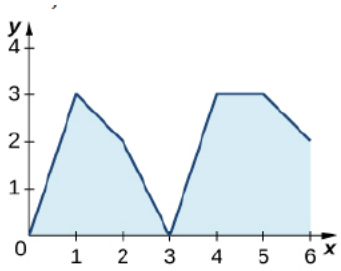
Part B. Find the total area between the pieces of the graph and the x axis.

Question 2. Use the fact that an integral is a limit of a summation, along with summation and limit laws, to prove the addition property of integrals, i.e., that

$$\int_a^b f(x) + g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

Question 3. (Exercise 160, parts a and b, in section 5.3 of Openstax *Calculus, Volume 1 for SUNY Geneseo.*)

Given a graph of $y = \int_0^x f(t)\,dt$ for some piecewise constant function f, find the intervals over which f is positive, negative, and zero; also find the minimum and maximum values of f. The graph from the textbook is



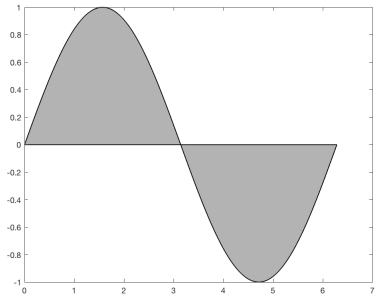
Question 4. (Based on Exercise 180 in section 5.2 of Openstax *Calculus, Volume 1 for SUNY Geneseo.*)

Evaluate

$$\int_1^4 \frac{2 - \sqrt{t}}{t^2}\,dt$$

Then use Mathematica to check your answer.

Question 5. Find the area between the x axis and one cycle of a sine wave, i.e., the shaded area in this plot:



Also use Mathematica to check your answer.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.