Math 221 06
Prof. Doug Baldwin

# Problem Set 11 — Summations

Complete by **Tuesday, November 12**
Grade by **Friday, November 15**

## Purpose

This problem set mainly helps you get familiar with summation (Sigma) notation and related algebra, with an introduction to definite integrals as sums. It also gives you a chance to practice using L'Hospital's rule. By the time you finish this problem set you should be able to ...

- Use L'Hospital's rule to find limits
- Translate between Sigma notation and explicit sums of numbers
- Use algebra and closed form formulas to simplify and evaluate summations
- Use Riemann sums to find approximate and exact values of definite integrals.

## Background

L'Hospital's rule is discussed in section 4.8 of our textbook, and we discussed it in class on November 6. Summations and their connection to definite integrals are in section 5.1 and the beginning of 5.2. We discussed them in class on November 7 and 8.

## Activity

Solve the following problems:

**Question 1.** (Exercise 356 in section 4.8 of Openstax *Calculus, Volume 1 for SUNY Geneseo.*)
Find the limit

$$\lim_{x \to \infty} \frac{e^x}{x}$$

**Question 2.** Find the limit

$$\lim_{x \to 0} \frac{x^3}{\sin^2 x}$$

**Question 3.** Use algebraic rules and closed form formulas for summations to evaluate the sum

$$\sum_{i=1}^{4} \frac{i}{2}$$

Also write the sum as an explicit sum of numbers and evaluate it, to confirm that you get the same result as algebra and formulas give (you may use a calculator or Mathematica to evaluate the explicit sum if you wish)

**Question 4.** (Based on problem 10 in section 5.1 of OpenStax *Calculus, Volume 1.*) Use algebraic rules and closed form formulas for summations to evaluate the sum

$$\sum_{j=11}^{20} (j^2 - 10j)$$

Also write the sum as an explicit sum of numbers, although you don't have to evaluate this explicit sum. You may use a calculator for the purely numeric parts of this problem, but not to simplify the sum.

**Question 5. Part A.** Approximate the definite integral

$$\int_1^2 \frac{1}{2} x \, dx$$

Using a Riemann sum with 100 intervals. Use a right-endpoint approximation. (I recommend that you use closed form formulas and algebraic rules for summations to simplify the Riemann sum rather than just trying to evaluate it by brute force, although you can use a calculator for the final numeric calculations.)

**Part B.** Find the exact value of the definite integral in Part A by taking the limit of the Riemann sum from that part.

**Part C.** Finally, find the exact value of the definite integral from Part A by using area formulas from geometry. (Consider plotting $y = \frac{1}{2}x$ over the interval $1 \leq x \leq 2$ if you want ideas about what formulas to use.)

## Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.