Math 221 03 — Final Exam

May 16, 2019

General Directions. This is an open-book, open-notes, open-computer test. However, you may not communicate with any person, except me, during the test. You have the full exam period (200 minutes) in which to do the test. Put your answer to each question in the space provided (use the backs of pages if you need more space). Be sure to *show your work!* In situations where you use Mathematica or other technology to calculate part of an answer, showing your work includes identifying the places you used technology and showing or describing the commands/functions you used. I give partial credit for incorrect answers if you show correct steps leading up to them; conversely, I do not give full credit even for correct answers if it is not clear that you understand where those answers come from. Good luck.

This exam contains 9 questions on 8 pages.

Question 1. (10 points) Phineas Phoole claims to know of a function $g(x)$ that has the following properties:

1. $g(0)$ is a local minimum
2. $g'(0)$ is undefined
3. $g''(x) < 0$ for $x < 0$
4. $g''(x) < 0$ for $x > 0$

Either sketch a graph of such a $g(x)$ over the interval $-1 \leq x \leq 1$, or explain why no such function can exist.

Question 2. (15 points) Consider finding the antiderivative

$$\int 4\sin(2x)\cos(2x)\,dx$$

There are at least two substitutions you could use to find this antiderivative. What are they? Show each by giving its "u" and "du" parts, and then evaluate the antiderivative using each.

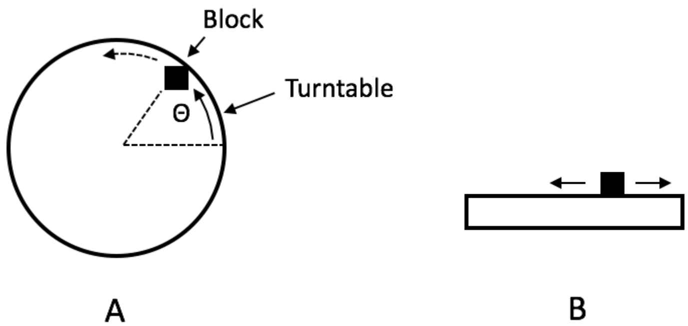
For up to 2 points of extra credit, explain why the two apparently different antiderivatives you get in answering this question both really are antiderivatives of $4\sin(2x)\cos(2x)$.

Question 3. (10 points) Find the value of b such that

$$\int_1^b 6x^2\, dx = 14$$

Question 4. (15 points) Imagine water flowing into a reservoir at a variable rate $w(t)$ gallons per minute, where t represents time. Assume the reservoir is initially empty, and that no water flows out of it. Explain how you could use a Riemann sum to model the volume of water in the reservoir.

Question 5. (15 points) Imagine a small block sitting on a rotating disk (e.g., on a lazy Susan or turntable). Seen from above, the block moves in a circle; after moving for t seconds, it has rotated to angle Θ radians from its original position, as shown in Part A in this picture:



However, as seen from the side the block simply seems to move back and forth, as seen in Part B of the picture. The block's apparent position left or right of the center of the disk is proportional to $\cos\Theta$.

Assume that such a disk and block system is set up so that the block as viewed from the side has position $x = 4\cos\Theta$ inches from the center of the disk — positive distances are to the right of the center, negative to the left. Furthermore, suppose that after turning for t seconds, the turntable has rotated a total of $\Theta = 6t$ radians. Find a function of t that tells you how fast the viewer from the side sees the block moving back and forth.

Question 6. (15 points) The designers of a hypothetical Mall of Geneseo have decided to put an arched window at one end of their mall. The upper edge of the window will follow the curve $y = 16 - x^4$, where y is height in feet. Assuming the bottom of the window is level at $y = 0$, what is the total area of the window?

Question 7. (15 points) Imagine a square that is expanding in such a way that its area increases by 2 square meters per second. How fast is the length of its diagonal increasing when the area is 8 square meters?

Question 8. (10 points) Izzy Newton, Sir Isaac's umpteenth descendant, has yet another of his historic but damaged notes. This one deals with limits, and reads…

We can find

$$\lim_{x\to\infty} \frac{5x^2}{2x^2+x-7}$$

by noticing that

$$\begin{aligned}
\lim_{x\to\infty} \frac{5x^2}{2x^2+x-7} &= ??? \\
&= \lim_{x\to\infty} \frac{5}{2+\frac{1}{x}-\frac{7}{x^2}} \\
&= \frac{5}{2}
\end{aligned}$$

Fill in an expression that could plausibly go in the missing part (i.e., the "???") of this example.

Question 9. (15 points) A totally fictitious psychologist claims that the average college professor's daytime alertness is described by the equation

$$A = \frac{3t}{t^2 + 4}$$

where A is alertness (in "lerts," an utterly made-up unit) and t is the number of hours since the professor got out of bed in the morning. Find the maximum and minimum alertness, in lerts, of the average professor during a 12-hour day (i.e., over the interval $[0, 12]$).

Have a Good Summer!