

Series
Real Analysis
Review Questions

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Disclaimer: This is a list of questions to guide you through your studies. Not everything that is asked in these questions will actually be tested (due to time constraints), and conversely, there might be a question that is tested that was not explicitly covered by these questions. Use these questions only as a supplement to the questions in the lecture notes/homework and don't feel like you need to answer every question here to be 100% ready for your test. **Solutions to these questions will not be provided.**

1. A series $\sum x_n$ converges if _____.
2. Prove that if $\sum x_n$ converges then $\lim_{n \rightarrow \infty} x_n = 0$.
3. Does the series $\sum_{n=1}^{\infty} \frac{n}{2n+1}$ converge?
4. Let (x_n) be a sequence of non-negative numbers. Prove that the series $\sum x_n$ converges if and only if the sequence (s_k) of partial sums is bounded. In this case, what is the value of $\sum x_n$?
5. Prove that the Harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.
6. Prove that if $|r| < 1$ then the Geometric series $\sum_{n=1}^{\infty} r^n$ converges. What is the value of $\sum_{n=1}^{\infty} r^n$?
7. Let (x_n) and (y_n) be non-negative sequences and suppose that $x_n \leq y_n$ for all $n \in \mathbb{N}$. Prove that if $\sum y_n$ converges then $\sum x_n$ converges.
8. Is the series $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$ convergent?
9. Suppose that $0 < x_n \leq 1$ for all $n \in \mathbb{N}$ and suppose that $\sum x_n$ converges. Is it necessarily true that $\sum x_n^2$ converges? Either prove that it is true or give a counterexample.
10. Suppose that $0 < x_n \leq 1$ for all $n \in \mathbb{N}$ and suppose that $\sum x_n$ converges. Is it necessarily true that $\sum \sqrt{x_n}$ converges? Either prove that it is true or give a counterexample.