

Disclaimer: This is a list of questions to guide you through your studies. Not everything that is asked in these questions will actually be tested (due to time constraints), and conversely, there might be a question that is tested that was not explicitly covered by these questions. Use these questions only as a supplement to the questions in the lecture notes/homework. **Solutions to these questions will not be provided.**

1. THE REAL NUMBERS AND THE COMPLETENESS PROPERTY

1. Prove that $\sqrt{2}$ is irrational.
2. Prove that if $a < x < b$ and $a < y < b$ then $|x - y| < b - a$.
3. State the Triangle inequality.
4. Let $f(x) = 3\sin(x) + x^4 - 7x$. Prove that $|f(x)| \leq 33$ if $-2 \leq x \leq 2$.
5. Give the definition for bounded above, bounded below, and bounded for a given non-empty set $S \subset \mathbb{R}$.
6. Give the definition of the supremum and infimum of a given set.
7. State the Completeness Property of \mathbb{R} .
8. Let A and B be non-empty. If $A \subset B$ and B is bounded above, prove that $\sup(A) \leq \sup(B)$.
9. Let A and B be non-empty and bounded above. Prove that $\sup(A + B) = \sup(A) + \sup(B)$.
10. Let A and B be non-empty and bounded above. Prove that $\sup(A \cup B) = \max\{\sup(A), \sup(B)\}$.
11. Let A and B be non-empty and bounded below. Prove that $\max\{\inf(A), \inf(B)\} \leq \inf(A \cap B)$.
Give an example of sets A and B where $\max\{\inf(A), \inf(B)\} < \inf(A \cap B)$.
12. Let A be non-empty and bounded below. Let $-A = \{y \in \mathbb{R} \mid y = -x, \text{ for some } x \in A\}$. Prove that $-A$ is bounded above and that $\sup(-A) = -\inf(A)$.
13. What is the Archimedean property of \mathbb{R} .
14. What is the Density theorem?
15. What is the Nested Intervals Property?