Available online at www.sciencedirect.com
SCIENCE DIRECT®

Chaos, Solitons and Fractals 25 (2005) 265–275

CHAOS
SOLITONS & FRACTALS

www.elsevier.com/locate/chaos

Wavelet-based prediction of oil prices

Shahriar Yousefi [a], Ilona Weinreich [b,*], Dominik Reinarz [b]

[a] *The Econometric Group, Department of Economics, University of Southern Denmark, DK-5230 Odense M, Denmark*
[b] *Department of Mathematics and Technology, University of Applied Sciences Koblenz, RheinAhr Campus, D-53424 Remagen, Germany*

Accepted 23 November 2004

Abstract

This paper illustrates an application of wavelets as a possible vehicle for investigating the issue of market efficiency in futures markets for oil. The paper provides a short introduction to the wavelets and a few interesting wavelet-based contributions in economics and finance are briefly reviewed. A wavelet-based prediction procedure is introduced and market data on crude oil is used to provide forecasts over different forecasting horizons. The results are compared with data from futures markets for oil and the relative performance of this procedure is used to investigate whether futures markets are efficiently priced.

1. Introduction

There are two types of trade in oil markets namely one that is based on immediate delivery and one that is based on delivery at some future point in time. Spot markets are dealing with the first type of trade while futures and forward markets are dealing with the other. The market conditions (e.g. market risk, trade imbalances, etc.) alongside a series of other factors (e.g. credit risk, insurance risk, seasonal factors etc.) are often blamed for causing inherent uncertainties in oil markets. Futures (and forward) markets are devised to provide hedging mechanisms for dealing with these uncertainties. In order to do so, the oil future contracts establish prices for buying or selling oil at future dates for given prices and in accordance to specific delivery and trade terms. In a way, future prices reflect the markets' expectations about the future market conditions and therefore the gap between the spot and future prices is often used as a signal for describing the general market condition. Various theories of Backwardation (e.g. when spot prices exceed future prices) or Contango (e.g. the inverse scenario) in literature are devised to address this issue from a mathematical perspective. These theories are based on the general concept of market efficiency and often conceive the future markets as (analytically) rational and efficient. Scholars often question whether these perceptions are indeed self-evident or, if they can be empirically verified in a convincing way. In particular whether futures markets are efficiently priced remains as a controversy. In this paper we intend to illustrate an application of wavelets methodology as a possible vehicle for addressing this issue. In short, the purpose of this paper is two-folded and includes a brief introduction to the wavelet methodology and a wavelet-based prediction procedure followed by an application of this procedure to market data on crude oil and a comparison of the results with data from oil futures market.

* Corresponding author. Tel.: +49 2642 932 217.
E-mail address: weinreich@rheinahrcampus.de (I. Weinreich).

0960-0779/$ - see front matter
doi:10.1016/j.chaos.2004.11.015

Although wavelets are known to have significant impact and widespread application in a variety of scientific fields (such as hydrodynamics, geophysics, data processing, image compression, detection of discontinuities, neural networks, etc.), the wavelet methodology appears to be an uncharted territory in the realm of social sciences. Only recently, a few studies rely on the wavelet methodology as a viable tool for studying dynamic properties of different financial and economic phenomena.

The origins of wavelet methodology can be traced to the classic theory of harmonic analysis and the seminal contributions of Joseph Fourier, Alfred Haar and Paul Levy. Since the appearance of the pioneering work of Morlet and Grossman in 1980s [1] wavelet methodology has been introduced to the literature as a regular alternative for analysing irregular situations where the data/signal contains scaling properties, discontinuities, sharp spikes etc. (see [2] for details). These contributions were followed by the introduction of the general idea of multi-resolution analysis by Mallat and Meyer and the notion of orthogonal wavelet bases by Daubechies in 1980s [3–5]. Fortunately, the conceptual framework of multi-scale analysis and discrete wavelet transformations seems to have the potential to become a useful vehicle for exploring and understanding various dynamic features of economic time series. As a matter of fact, during the last ten years or so, a few wavelet related studies of financial forecast strategies have surfaced in the literature (see [6–9]). Ramsey [10] provides a general review of wavelet literature in economics and finance and puts a few interesting contributions in perspective. Alongside other issues, Ramsey also considers a few problems and potentials associated with the application of wavelets for forecasting economic/financial series. Our study is partly motivated by these considerations.

In this paper we intend to rely on wavelet methodology and investigate whether if future contracts are effectively priced. In doing so, we focus on the usefulness and performance of wavelets in providing out of sample forecasts for the oil prices. We also try to deal with several associated issues and provide a balanced account of the problems and promises. The applied procedure is motivated by some basic properties of wavelets and is based on the application of the discrete wavelet transform (DWT) on time series of average monthly crude oil prices. The idea is to subdivide the price data/signal in low and high frequency part. By relying on DWT, the data is decomposed in several scales and coarse and fine parts of the data are obtained. The coarse scales reveal the trend, while the finer scales might be related to seasonal influences, singular events and noise. This is followed by an appropriate and adaptive extension of the signal (which is depending on the behaviour on each scale). Consequently, (out of sample) forecast values on each scale are calculated and the inverse wavelet transform is used to generate a forecast for the whole signal. The forecasted values are calculated for 1, 2, 3 and 4 months ahead. In order to illustrate the predictive power of this procedure, the forecast procedure is repeated for a large number of samples with identical lengths. The resulted forecasted values are then compared with the actual market expectations (represented by the average monthly future prices) for the same periods.

The paper develops as follows. Section 2 provides the historical background and focuses on the general features of wavelets. Section 3 provides an analytical introduction to the notion of multi-scale analysis and elaborates on discrete wavelet transforms in wavelet theory. A brief review of the role of wavelets in economic literature is presented in Section 4. A detailed specification of the wavelet-based forecasting procedure alongside details on data and a balanced presentation of the obtained results are provided in Section 5. Section 6 provides the concluding remarks and a general discussion of the results.

2. Roots in harmonic analysis and the role of scaling properties

The Fourier transform technique is a fairly standard tool for analysing harmonic and stationary time series. The method is based on a linear decomposition of the signal into Fourier bases (i.e. sine and cosine functions of different frequencies) while the properties of the underlying function are supposed to be derived from the properties of the associated trigonometric bases. In other words, the embedded information in a given time series is represented as a function of frequency with no specific mechanism for preserving information in time. Such an approach seems to be useful for the analysis of well-behaving stationary or periodic signals but highly problematic for dealing with local phenomena such as singularities and sharp transients.

Around 20 years ago researchers started to use wavelets as an alternative to Fourier transform for the analysis of acoustic and seismic signals that usually pose singularities and irregular transients [1]. In contrast to regular Fourier transforms (e.g. various manifestations of extended trigonometric functions), wavelet analysis works with translates and dilates of a single local function ψ: the mother wavelet. An advantage of the wavelet transform in comparison with the Fourier transform is that the wavelet coefficients reflect in a simple and precise manner the properties of the underlying function. A mother wavelet is locally defined, i.e. it either should have compact support or decay sufficiently fast. It implies that small changes in the signal have only limited influence on the coefficients of the corresponding wavelet representation. The existence of mother wavelets (with compact support and certain smoothness) is not a trivial analytical issue and details concerning the construction of a class of compactly supported wavelets with arbitrary smoothness were first addressed by Daubechies in the late 1980s [11]. Daubechies' work provides a reliable vehicle for obtaining orthogonal wavelet bases by translating and dilating the mother wavelet. A mother wavelet ψ with compact support is located in a finite interval. It implies that a singularity can be analysed by considering only those translates of ψ which overlap the singularity. Finer details can be analysed by scaled versions of ψ with smaller support. Consequently, the local analysis of a function is possible with the aid of only few basis functions. On the other hand by using wavelets we obtain a decomposition into scales of different resolution (the so called multi-scale decomposition).

At this stage, it is worth to note that in wavelet literature the notion of scaling is somehow different to alternative views that have already been introduced and employed in economics and finance literature. As a matter of fact, there is a sizeable body of literature on scaling phenomena in nature and society. Depending on the nature of the processes, different scenarios are occasionally formulated as scaling laws or presented in the framework of the conditional predictive distributions. A large segment of literature on scaling, deals with issues related to economics and finance (see Ref. [12]). In this tradition, the basic notion of scaling is often associated with the presence of deterministic or statistical regularities (in a given phenomenon) that are (seemingly) independent of the scale details. This notion is often illustrated by considering the presence of common properties in the plots of one variable against another variable in logarithmic scale. Such common properties have been regularly addressed in theoretical and empirical studies of economic data. Among the most well-known examples are Pareto's law (on income distribution), Zipf's law (on city size etc.), Gibrat's Law (on distributions of firm size and growth), Mandelbrot's studies of commodity prices and various versions of Bak's sand pile model.

Despite the initial sceptical reaction to these contributions, the work on scaling relations in economics is fairly justified. Obviously, the presence of scaling relations in a given economic process signifies empirical regularities that are expected to be considered, described, interpreted and finally replicated in any reasonable modelling effort. As far as the analysis of economic time series (e.g. commodity prices) are concerned, the presence of scaling relations can be used to characterize the statistical properties of the underlying process and to provide alternative means for dealing with the volatility issue and other issues related to conditional moments (mean, variance, etc.). A fairly known example is the power law distribution of price increments in Mandelbrot's studies of commodity prices. In this case, the underlying probability structure is postulated to follow a relation similar to $\Pr(I > i) \simeq i^{-\alpha}$. The standard restriction is to consider $\alpha < 2$ for independent increments. An alternative model, involving multi-fractals and dependent increments allows α to be between 1 and ∞. Mandelbrot's recent contributions to the first issue of the Quantitative Finance elaborate on these models and this particular perception about the general issue of scaling in financial prices. Furthermore, the interested reader is advised to consult Brock [12] for detailed elaborations on these issues and for further references to the literature on scaling in economics.

As regards the present study, we intend to confine our efforts to a general application of multi-scale analysis as it is understood in wavelet literature. The basic idea is to consider a signal which can be decomposed by wavelet transform in different scales. The scales contain contributions of the signal of different frequencies. When embedded in an appropriate function space, the multi-resolution (or multi-scale) analysis of a function (or signal or time series) can be performed.

3. Wavelets in economics and finance

In this section, we intend to provide a short overview of a few wavelet related contributions in the economic literature. It is worth to mention that Ramsey [10] and Gencay et al. [13] are two other sources that provide broad reviews of the literature. In order to maintain brevity, we only refer to contributions that are not mentioned in these sources. The interested reader is well-served by consulting these sources for further details.

The application of wavelet theory in modelling and analysing economic data (and phenomena) is still in its infancy and many properties of these models are not explored yet in economic and finance literature. Apparently the upsurge of interest in application of wavelets in economic literature began in the middle of 1990s by the contributions of Ramsey and his collaborators. Seemingly, the first paper that appeared in 1995 focuses on the scaling properties of speculative assets as a possible mechanism for revealing the underlying hidden order in the data [14]. To the best of our knowledge a paper by Cao et al. [6] seems to be the first paper that actually combines the wavelet networks with Taken's phase space reconstruction technique in order to devise a procedure for forecasting economic time series.

Wong et al. [15] provide a recent example of using wavelet-based methods for forecasting exchange rates. Three other regular time series methods are used as references in order to justify the application of the suggested forecasting procedure. Davidson et al. [16] use wavelets in order to introduce a semi parametric approach for analysing commodity prices. They conclude that wavelet analysis is particularly useful in describing the general features of the commodity prices such as structural breaks, co-movements of prices, and the unstable variance structure and time dependent volatility. Macroeconomics is another interesting area. A recent application of wavelets in analysing economic cycles is provided by Jagric [17]. Almasri and Shukur [18] address the causal relation between spending and revenue. Ramsey [10] and Gencay et al. [19] look into similar relations between Money and Income, Expenditure and Income and Money Growth and Inflation. Tkacz [20] builds a bridge to co-integration literature and proposes to use wavelet OLS estimators for estimating fractional integration of US and Canadian interest rates.

Finally, it is also worth to mention a recent stream of the papers that are predominantly using wavelets to address theoretical econometric issues such as devising test (and/or estimation) routines for significance, variance structures, serial correlation, model specification and memory processes (see for instance Refs. [21–28]). These papers are quite dispersed in the literature and their actual content might be out of the scope of this work. Nevertheless, we find it appropriate to credit them and recommend the interested reader to consult them for further details.

4. Multi-scale analysis and discrete wavelet transform

This section provides a schematic overview of the basic mathematical issues and the associated details. The underlying mathematical structure for wavelet bases of a function space V is a multi-scale decomposition (the so-called multiscale analysis). The classical example from Meyer [5] considers such a decomposition for the Hilbert space $L_2(R)$. Multiscale analysis involves a hierarchical sequence of nested subspaces V_j of the function space V (i.e. $\cdots \subset V_j \subset V_{j+1} \subset \cdots$) with empty intersections and dense closures in $L_2(R)$. It is a decomposition in several resolution levels that requires a two scale relation such as $f(x) \in V_j \iff f(2x) \in V_{j-1}$.

Starting point for such a decomposition (and for the construction of the wavelet bases) is the so-called father wavelet φ. This function should preferably have compact support and its integer translates $\varphi(x-k)$, $k \in Z$, span a space V_0. A finer space V_j is spanned by the integer translates of the scaled functions $\varphi(2^j x - k)$. Scaling by 2^j provides the basis functions for the space V_j and the nestedness of the spaces V_j yields a scaling equation

$$\varphi(x) = \sum_{k \in Z} a_k \varphi(2x - k) \tag{1}$$

with appropriate coefficients a_k, $k \in Z$. The mother wavelet ψ is obtained by building linear combinations of the scaled father wavelet. In order to maintain orthogonality one has to select the proper coefficients in that linear combination. In other words one has to secure that the translates of father and mother wavelet are orthogonal:

$$\langle \varphi(\cdot - k), \psi(\cdot - l) \rangle = 0, \quad l, k \in Z. \tag{2}$$

The scaling equation (1) and the orthogonality condition (2) lead to conditions on coefficients b_k which characterize a mother wavelet as a linear combination of the scaled and dilated father wavelets φ:

$$\psi(x) = \sum_{k \in Z} b_k \varphi(2x - k). \tag{3}$$

Actually, the wavelets ψ are uniquely determined by their coefficients $\{b_k\}k \in z$. A common example for a (mother) wavelet is the Haar wavelet ψ with

$$\psi(x) = \begin{cases} 1 & \text{for } x \in [0, 0.5], \\ -1 & \text{for } x \in [0.5, 1], \\ 0 & \text{for } x \notin [0, 1], \end{cases}$$

that can be obtained from the father wavelet

$$\varphi(x) = \begin{cases} 1 & \text{for } x \in [0, 1], \\ 0 & \text{for } x \notin [0, 1]. \end{cases}$$

Note that in this case the father wavelet satisfies the scaling equation $\varphi(x) = \varphi(2x) + \varphi(2x - 1)$, so the coefficients in (1) are $a_0 = a_1 = 1$ and $a_k = 0$ for $k \neq 0, 1$. The Haar wavelet is defined as a linear combination of the scaled father wavelets $\psi(x) = \varphi(2x) - \varphi(2x - 1)$ which means the coefficients b_k in (3) are $b_0 = 1$ and $b_1 = -1$ and $b_k = 0$ otherwise. Haar wavelets can be interpreted as a simple Daubechies' wavelet of order 1 with 2 coefficients. In general, Daubechies' wavelets of order N are not given analytically but described by $2N$ coefficients. The higher N, the smoother the corresponding Daubechies' wavelets are (the smoothness is around $0.2 * N$ for greater N). Daubechies' wavelets are constructed in a way such that they give rise to orthogonal wavelet bases. By combining ideas from Fourier analysis and filter banks it is possible to describe conditions for orthogonality of the bases with respect to conditions for the filters a_k and b_k in Eqs.

(1) and (3). It is possible to show that the orthogonality of the translates of φ and ψ, requires that $\sum_k a_k = 2$ and $\sum_k b_k = 2$. In the Haar case this can be easily checked.

Given a fixed wavelet, the whole wavelet basis is obtained by scaling and translating the function ψ. The scaled and translated (and normalized) versions of ψ are denoted by

$$\psi_{j,k} = 2^{j/2}\psi(2^j x - k).$$

With a (orthogonal) wavelet ψ the set $\{\psi_{j,k}|j, k \in Z\}$ is an orthogonal wavelet basis. A function f can be represented as

$$f = \sum_{j\in Z}\sum_{k\in Z} c_{j,k}\psi_{j,k}(x).$$

The discrete wavelet transform (DWT) corresponds to the mapping $f \to c_{j,k}$. DWT provides a mechanism to represent data or time series f in terms of coefficients that are associated with particular scales (see Ref. [2]) and therefore is regarded as a fairly effective instrument for signal analysis. The decomposition of a given signal f into different scales of resolution is obtained by the application of the DWT to f. In practical applications, we only use a small number of levels j in our decomposition (for instance $j = 4$ corresponds to a four level wavelet decomposition of f).

The first step of DWT corresponds to the mapping f to its wavelet coefficients and from these coefficients two components are recovered namely a smoothed version (the so-called approximation) and a second component that corresponds to the deviations or the so-called details of the signal. A decomposition of f into a low frequency part a_1 and a high frequency part d_1 is represented by $f = a_1 + d_1$. The same procedure is performed on a_1 in order to obtain a decomposition in finer scales; $a_1 = a_2 + d_2$. A recursive decomposition for the low frequency parts follows the directions that are illustrated in the following diagram:

$$\begin{array}{ccccccccc} f & \cdots & a_1 & \cdots & a_2 & \cdots & a_3 & \cdots & a_N \\ & \ddots & & \ddots & & \ddots & & & \\ & & d_1 & & d_2 & & d_3 & \cdots & d_N \end{array}$$

The resulting low frequency parts $a_1, \ldots, a_N$ are approximations of f, and the high frequency parts $d_1, \ldots, d_n$ contain the details of f. This diagram illustrates a wavelet decomposition into N levels and corresponds to

$$f = d_1 + d_2 + \cdots + d_{N-1} + d_N + a_N.$$

In practical applications, such a decomposition is obtained by using a specific wavelet. Several families of wavelets have proven to be especially useful in various applications. They differ with respect to orthogonality, smoothness and other related properties such as vanishing moments or size of the support. In the present case it turned out that Daubechies' wavelets yield good results.

5. Wavelet-based forecasting

This section provides a schematic overview of a wavelet-based forecasting procedure. It is followed by an application of this procedure on averaged monthly data from the oil market. The data material consists of averaged monthly WTI spot prices ($/bbl) alongside NYMEX futures prices for future contracts ($/bbl) with 1, 2, 3 and 4 months' horizon. The original data material stems from the website of US Department of Energy, EIA and covers the period 2 January 1986 to 31 January 2003. The data on spot prices is used to generate out of sample forecast for oil prices in 1, 2, 3 and 4 periods ahead. Since the NYMEX oil future prices are generally regarded as global benchmarks, the data on NYMEX future contracts is used to form a mechanism for comparing own forecasting results with market expectations. These results are used to control if the oil futures market is effectively priced.

The entire procedure consists of four steps namely pre-processing of data, wavelet decomposition, signal extension and wavelet reconstruction. Pre-processing of data addresses various inconsistencies and missing points in the data. These problems are often related to certain days in which the market has been closed (weekends and national holidays), or the days in which the trade has been halted due to unexpected events and crises. As a matter of fact, the number of rows in the two underlying data sets (for the period 2 January 1986 to 31 January 2003) is not the same. Obviously, this kind of problems, make it fairly difficult to use the daily data from futures markets as a basis for comparison. As an alternative, we suggest to generate the monthly average of both types of data and use them as input for further analysis. This approach provides a total of 216 consecutive monthly observations for averaged spot prices and averaged prices for 1, 2, 3 and 4 months future contracts. The monthly averaged price data is used to select 100 random samples of consecutive observations with equal length. Each individual sample is used as an input for the proposed forecasting

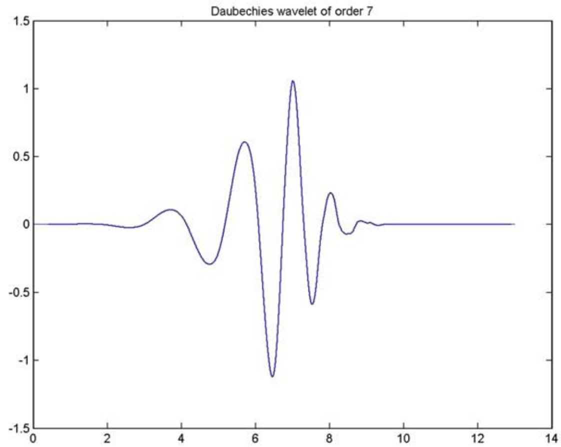

Fig. 1. Daubechies wavelet of order 7.

procedure. In each step, the out of sample forecast for 1, 2, 3 and 4 periods ahead are calculated. Obviously, this approach implies some degree of smoothing but (in balance) it also provides a reasonable basis for a repeating execution of the proposed forecasting procedure. The Wavelet toolbox in Matlab is used as a standard tool for the process of wavelet decomposition. This step involves several different families of wavelets and a detailed comparison of their performance. In our case, the Daubechies' wavelets of order 7 (Fig. 1) outperform the other alternatives. A 5 level wavelet decomposition of the given times series $X_N =: f$ is performed

$$f = a_5 + d_5 + \cdots + d_2 + d_1.$$

Obviously, the smooth part of f is stored in a_5, and details on different levels are captured by $d_1, \ldots, d_5$. Consequently a decomposition of the time series in five different scales is obtained.

Fig. 2a illustrates the decomposition of the original signal. Here the first level of details provides the short term variation within a month or two, while the next levels of details represent the variations within 2^j months' horizon, indicating that the oil price dynamics is mainly caused by short term fluctuations.

Fig. 2b illustrate the decomposition of an arbitrary sample (from the first half of the signal). This is used to illustrate the underlying mechanism for the forecasting procedure since the actual forecasts are based on an appropriate extension of samples of similar type and size.

The idea is to use the wavelet transform and to extend the decomposed data on each level. It is obvious that the signal on the different levels illustrates a quite different behaviour. For the approximation level a_5 (and the highest detail level d_5), a spline fit is applied to extend the signal. The lower detail levels $d_1, \ldots, d_4$ show higher frequencies. Therefore a spline extrapolation would not be appropriate. Since the oscillations occur in the detail parts, a trigonometric fit seems to be more appropriate for obtaining an extension for those parts of the signal. A sine fit as described in [29] is applied to extend the decomposed signal on these levels. The Matlab routine sinefit.m by Chen [30] is used for this purpose. This routine provides optimal extraction of features in sampled noisy sinusoidal signals.

The reconstruction of the wavelet involves the application of the inverse wavelet transforms on the decomposed and extended signals on different scales. By doing so an extended signal that also includes the out of sample forecasts for 1, 2, 3 and 4 periods ahead is obtained.

This forecasting procedure is repeatedly applied to 100 random samples. Fig. 3 summarizes the results for forecasting 1 period ahead. The top diagram illustrates the plot of the forecasted values vs. the actual values. As an alternative, the second diagram illustrates the plot of the future values vs. the actual values. The Figs. 4–6 provide similar results for other forecasting horizons (e.g. 2–4 months ahead). The corresponding correlation data is given in Table 1. Next section puts these results in perspective and elaborates on a few related issues.

6. Discussion and concluding remarks

Present paper shares much of it's underlying analytical reasoning with the work recently reported in [31,32]. As regards the empirical findings, our results illustrate a persistent pattern (over 1–4 months' horizon) indicating that the

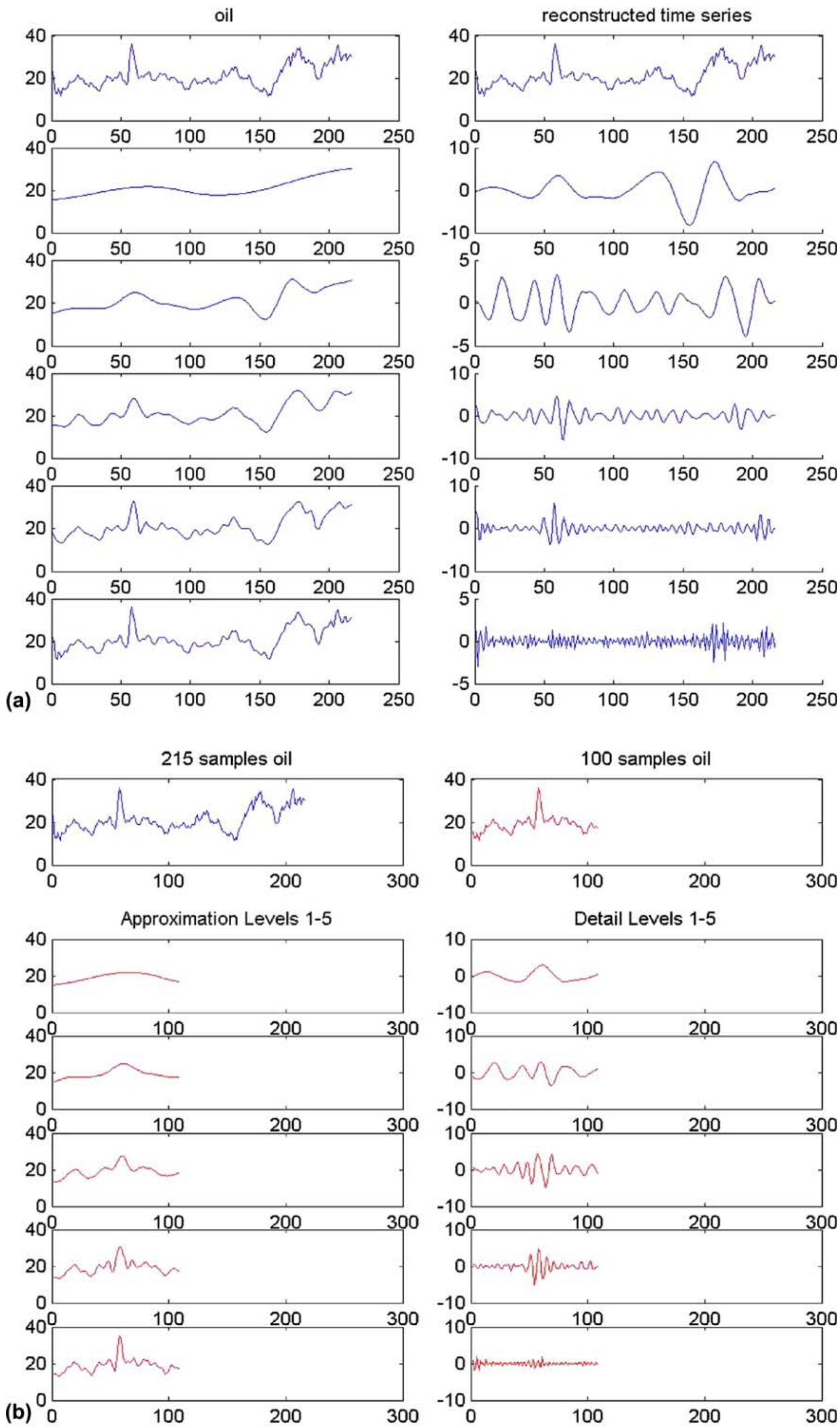

Fig. 2. (a) 5 level decomposition of the oil price data. (b) 5 level decomposition of an arbitrary sample (first half).

wavelet-based forecast procedure outperforms the future market in average. Apparently, this superiority in forecast does not diminish when the forecasting horizon is extended. This feature is particularly interesting, since one usually expects the opposite scenario (i.e. declining predicting power for extended forecasting horizons). Seemingly, these

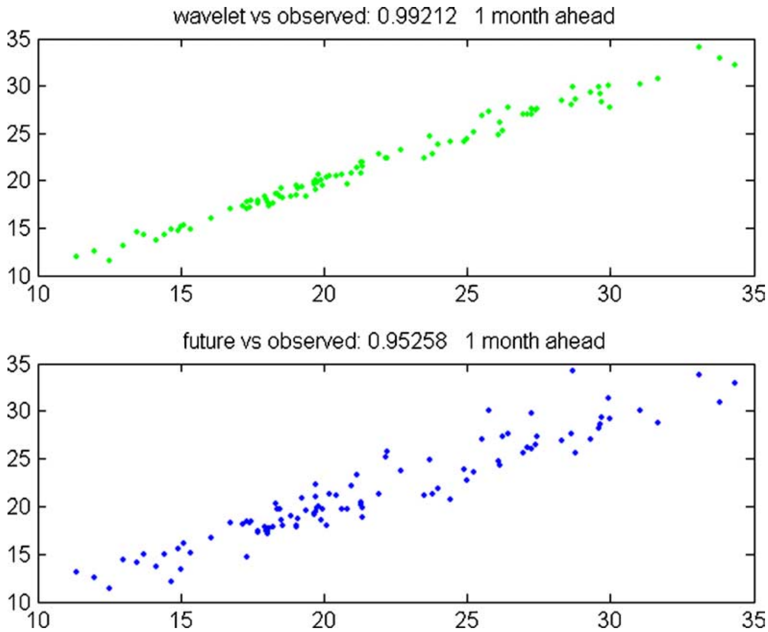

Fig. 3. Relative performance of wavelet-based forecast vs. future data (1 month).

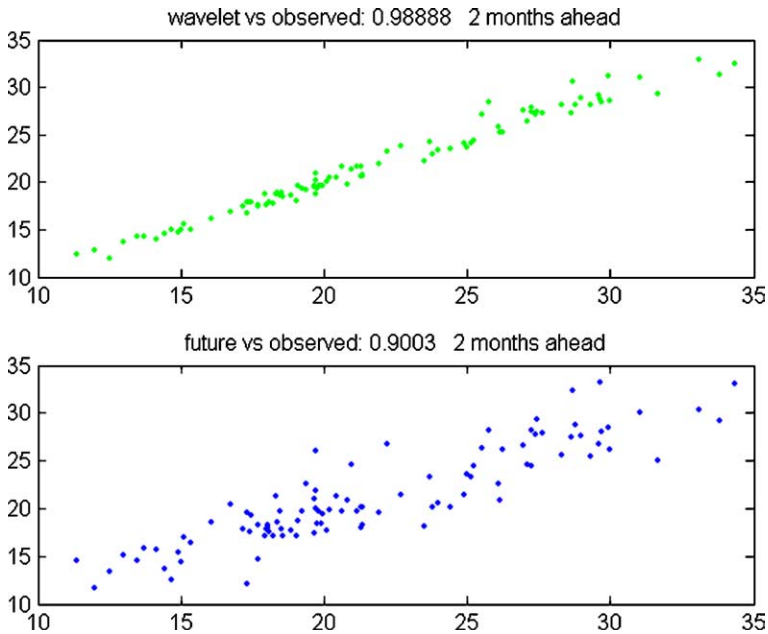

Fig. 4. Relative performance of wavelet-based forecast vs. future data (2 months).

results support the idea that the futures market might not be efficiently priced. Nevertheless, a balanced interpretation of these results requires further elaboration on a series of issues.

First, it is worth to note that the pre-processing of the data (and work with averaged monthly data) has removed the inner monthly variations from our analysis. On the other hand, the market for oil is known to be highly influenced by seasonal movements (that are often related to monthly variations in production and consumption) and to a limited degree to daily variations. Furthermore, transactions in oil spot markets involves actual delivery within three to four

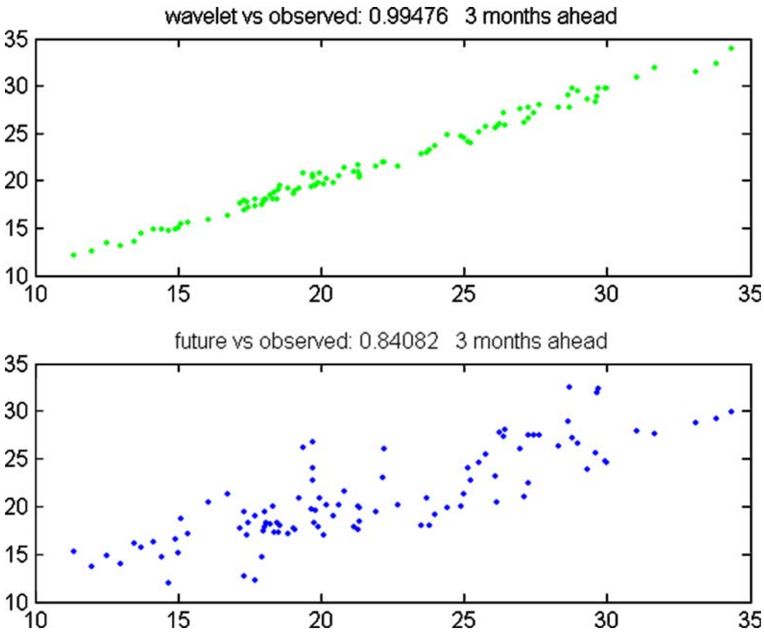

Fig. 5. Relative performance of wavelet-based forecast vs. future data (3 months).

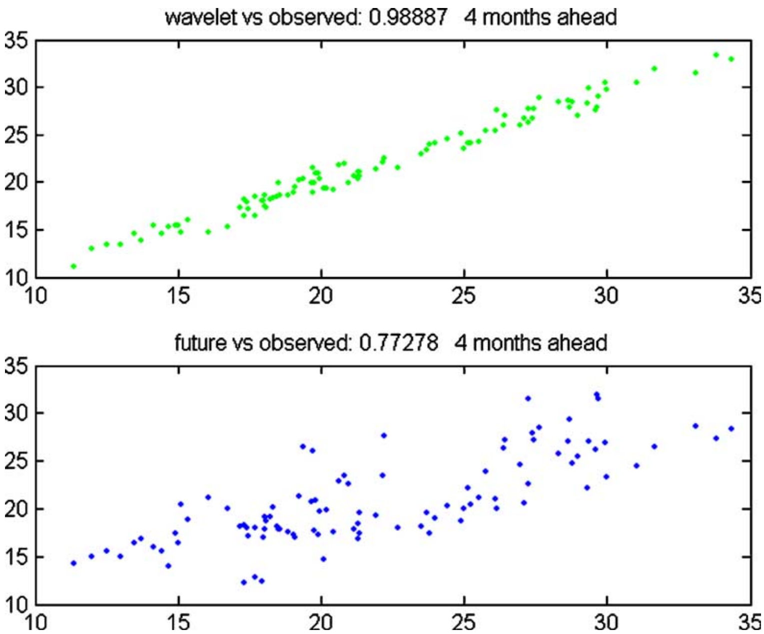

Fig. 6. Relative performance of wavelet-based forecast vs. future data (4 months).

weeks and therefore it is reasonable to assume that the dealers in NYMEX have a fair feeling about the supply/demand imbalances in short term. Actually, this feature has been captured in the three graphs associated with d_1, d_2, d_3 in the process of wavelet decomposition (Fig. 2), indicating that the oil price dynamics is mainly influenced by monthly and seasonal fluctuations.

Second, it is worth to mention that the predictive power of wavelet-based forecasting procedure is highly sensitive to sample size. In order to illustrate this feature, we have focused on one period ahead forecast and repeated the

Table 1
The correlation data and the relative performance of the two alternatives

Forecasting horizon	Wavelet-based forecast	Futures
1 month ahead	0.992	0.952
2 months ahead	0.998	0.903
3 months ahead	0.995	0.841
4 months ahead	0.998	0.772

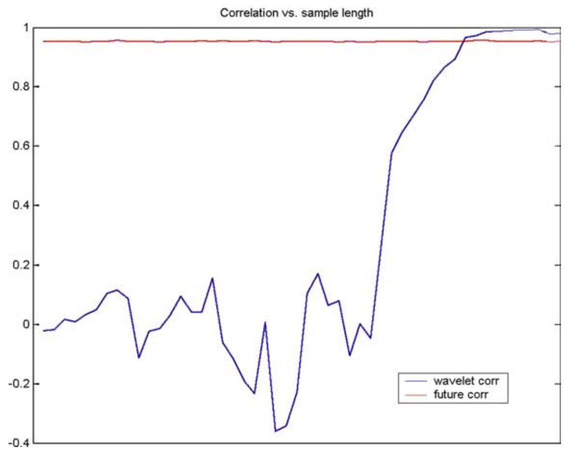

Fig. 7. Illustrating the sensitivity of wavelet-based forecast to sample size (1 months).

forecasting procedure for sample sizes between 60 and 110. In each case the correlation coefficient between the predicted values and the actual values is calculated and compared with the corresponding correlation coefficients between the future values and the actual values. Fig. 7 illustrates the predictive power of both alternatives for varying sample sizes. The calculated correlation coefficients are given on the vertical axe and the sample size on the horizontal axe. As illustrated in Fig. 7, the wavelet-based forecasting procedure works best for large sample sizes around 100 and larger. It is not surprising to expect similar patterns to emerge for other forecasting horizons as well. Wavelet-based estimation of the variance and covariance structure of the oil market (alongside the estimation of higher moments) and the general issue of sample sized related sensitivity of wavelet-based forecasting are interesting issues that are demanding separate studies.

References

[1] Grossmann A, Morlet J. Decomposition of hardy functions into square integrable wavelets of constant shape. SIAM J Math Anal 1984;15:723–36.
[2] Percival DB, Walden AT. Wavelet methods for time series analysis. Cambridge, MA: Cambridge University Press; 2000.
[3] Mallat S. A theory for multiresolution signal decomposition: the wavelet representation. IEEE Trans Pattern Anal Mach Intell 1989;11:674–93.
[4] Daubechies I. Ten lectures on wavelets. Philadelphia: SIAM; 1992.
[5] Meyer Y, Coifman R. Wavelets. Cambridge: Cambridge University Press; 1997.
[6] Cao L, Hong Y, Zhao H, Deng S. Predicting economic time series using a nonlinear deterministic technique. Comput Econom 1996;9(2):149–78.
[7] Aussen A, Campbell J, Murthagh F. Wavelet-based feature extraction and decomposition strategies for financial forecasting. J Comput Intell Finance 1998;6(2):5–12.
[8] Milidiu RL, Machado RJ, Rentera RP. Time-series forecasting through wavelet transform and a mixture of expert models. Neurocomputing 1999;28:145–6.
[9] Fryzlewicz P, Van Bellegem S, von Sachs R. Forecasting non-stationary time series by wavelet process modelling. Technical report, Université catholique de Louvain: Institut de Statistique, 2002.

[10] Ramsey JB. Wavelets in economics and finance: past and future. Stud Nonlinear Dynam Economet 2002;6(3):1–27.
[11] Daubechies I. Orthonormal bases of compactly supported wavelets. Commun Pure Appl Math 1988;41(7):909–96.
[12] Brock WA. Scaling in economics. Indus Corporate Change 1999;8(3).
[13] Gencay R, Selcuk F, Whitcher B. An introduction to wavelets and other filtering methods in finance and economics. San Diego, London and Tokyo: Academic Press; 2002.
[14] Ramsey DJB, Usikov, Zalavsky G. An analysis of US stock price behavior using wavelets. Fractals 1995;3(2):377–89.
[15] Wong H, Wai-Cheung I, Zhongjie X, Lui X. Modelling and forecasting by wavelets, and their application to exchange rates. J Appl Statist 2003;30(5):537–53.
[16] Davidson R, Labys WC, Lesourd J-B. Wavelet analysis of commodity price behaviour. Comput Econom 1998;11(1–2).
[17] Jagric T. Business cycles in central and east European countries. Eastern Europe Econom 2003;41(5):6–23.
[18] Almasri G, Shukur A. An illustration of the causality relation between government spending and revenue wavelet analysis on Finnish data. J Appl Statist 2003;30(5):571–84.
[19] Gencay R, Selcuk F, Whitcher B. Systematic risk and timescales. Quant Finance 2003;3(2):108–16.
[20] Tkacz G. Estimating the fractional order of integration of interest rates using a wavelet osl estimator. Stud Nonlinear Dynam Economet 2001;5(1).
[21] Hardle W, Sperlich S, Spokoiny V. Structural tests in additive regression. J Am Statist Assoc 2001;96(456):1333–47.
[22] Lee J, Hong Y. Testing for serial correlation of unknown form using wavelet methods. Economet Theory 2001;17(2):386–423.
[23] Hong J, Lee Y. One-sided testing for arch effects using wavelets. Economet Theory 2001;17(6):1051–81.
[24] Stengos Y, Sun T. A consistent model specification test for a regression function based on nonparametric wavelet estimation. Economet Rev 2001;20(1):41–60.
[25] Pan X, Wang Z. A wavelet-based nonparametric estimator of the variance function. Comput Econom 2000;15(1–2):79–87.
[26] Jensen MJ. An alternative maximum likelihood estimator of long-memory processes using compactly supported wavelets. J Econom Dynam Control 2000;24(3):361–87.
[27] Jensen MJ. An approximate wavelet MLE of short- and long-memory parameters. Stud Nonlinear Dynam Economet 1999;3(4):239–53.
[28] Fan S-K, Lin J. Test of significance when data are curves. J Am Statist Assoc 1998;93(443):1007–21.
[29] Dou YQ, Chen H. Optimal features extraction of noisy sinusoidal signals using two-stage linear least squares fitting. Technical report. Available from: <http://www.csois.use.edu/people/yqchen>.
[30] Chen YQ. Technical report. Available from: <http://www.csois.usu.edu/people/yqchen/sinefit.html>.
[31] Weinreich I, Rickert H, Lukaschewitsch M. Wavelet based time series prediction for air traffic data. Paper presented at SPIE International Symposium on Photonics Technologies for Robotics, Automation and Manufacturing, October 2003.
[32] Maaß P, Köhler T, Kalden J, Costa R, Parlitz U, Merkwirth C, et al. Mathematical methods for forecasting bank transaction data. Source Preprint Series DFG-SPP 1114. Preprint 24, 2002.